

A Bid-Rent Network Equilibrium Model

by
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Abstract

This thesis discusses the development of a new model for investigating the relationship between transport and the location of activities. The research consists of three stages. In the first phase, the structural features of the relationship, which are the characteristics of locations, the decision-making processes of households, and the interaction between transport and land-use, are identified. Existing approaches are reviewed using these three components. The review shows that no existing framework satisfactorily represents the requirements in modelling the relationship. Secondly, a bid-rent network equilibrium model is developed. The modelling is considered in terms of competition for locations. Difficulties in analysing the unique characteristics of locations, namely heterogeneity and indivisibility, are discussed. A hedonic interpretation is incorporated to overcome the difficulties. The model represents the decision-making processes of households using the framework of an n -player non-cooperative game. The Nash equilibrium for this game is defined. The game is accompanied by the systematic interactions between transport and land-use. A mutual adjustment process addresses these interactions. A bi-level mathematical programme is suggested to embody the three components. The resulting formulation is interpreted as an oligopolistic Cournot game, which is an approximation of the n -player non-cooperative game. The functional relationship between the decision variables of the upper and lower levels in the bi-level model produces endogenously-determined transport impedance and locational attractiveness. The model incorporates a multiclass framework to consider interclass interactions, which establishes a multiclass bid-rent network equilibrium model. A heuristic algorithm is provided for the solution-finding technique of the bid-rent network equilibrium model. The algorithm combines a path-based routine for calculating the equilibrium solution to the lower level with the Newton-Raphson procedure for estimating the parameters of the hedonic-based stochastic bid-rent function in the upper level. The operation of the algorithm is examined using simple numerical examples. The final stage is an application of the bid-rent network equilibrium model to a real network. A medium-sized city is chosen for the case study. The objective of the third stage is to demonstrate the ability of the model to investigate the relationship between transport

and the location of activities. A base run and two policy runs are simulated. The base run means a simulation conducted using surveyed data. The policy runs represents the introduction of a congestion charge and the release of land for housing development. Class specific spatial behaviour is obtained. The behaviour is demonstrated using network performance indices representing transport impedance and locational attractiveness. Some policy implications of the simulation are presented.

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1. Introduction

This study explores the development of a *bid-rent network equilibrium model* for representing the relationship between transport and the location of activities. This relationship, traditionally, has been regarded as one of the most important components in land-use and transport interaction studies. The interaction between transport and land-use is a two-way process. The process is more complicated than other reciprocal processes that are frequently encountered in every day life. This is mainly because the various interactions take place over different time scales and involve factors with varying degrees of certainty. Hence, an analysis of the interaction between transport and land-use requires disentangling diverse relationships among the factors and so is a difficult quest. One way to tackle this problem is to try to understand the basic mechanisms involved and to incorporate the components in a general framework. The relationship between transport and the location of activities is generally regarded as the foundation of the interaction between land-use and transport. Thus, an investigation into this relationship is a core task to appreciate the interaction with respect to the representation of the present and the future states of such systems.

Conventionally, four types of behaviour have been identified in this relationship (Wilson, 1970; Boyce and Southworth, 1979; Boyce, 1980; Nagurney and Dong, 2002b): households with fixed economic activity locations seeking residential locations; households with fixed residential locations seeking economic activity locations; households seeking both residential and economic activity locations; and households with fixed residential and economic activity locations. The first type of behaviour has been studied more actively than any of the other types. This is for the following several reasons. In the first place, residential land-use is more dominant than any other land-uses within a city (Alonso, 1965, p.2). This dominance means that an investigation into the residential land-use would offer a basic understanding of urban structure. Secondly, the residential location is the origin of the majority of travel demands. It has been found in practice that more than eighty percent of trips are generated in relation to the residence (Ortúzar and Willumsen, 1994, p.115). Thirdly, the model of the first type of behaviour can easily be converted into those of the other

types of behaviour. Extensions of the model for the first behaviour to the other cases are generally straightforward with slight modifications.

The nature of this problem has attracted attention for reasons other than those mentioned above. First of all, the relationship recognises the unique characteristics of locations. In conventional economic analyses, two assumptions about the features of goods are required, namely homogeneity and divisibility. Locations, however, violate these two requirements. Indeed, a location is a heterogeneous product. There is a substantial variation in the structures, the transport connections, the quality of local public services, the characteristics of neighbourhood, and other factors (Ellickson, 1981). Furthermore, a location is an indivisible product. There are observations on expenditure on locations, but imprecise references as to which components are purchased. There is no direct information on the prices of the components that are embodied. Thus, the process of production, exchange and consumption of each attribute is implicit. These unique characteristics require an alternative approach. Secondly, a complex decision-making process is considered in this problem. The decision-making unit for the selection of a residential location is a household. Each household normally consists of several members. The residential location may affect their patterns of activities. This means that the decision to select a residential location is a compromise that considers the locational requirements of all its members. Even though a decision has been made within a household, the location chosen is not always occupied by the household. This difference occurs because there are many potential competitors. This suggests that a household interacts with other households. Therefore, the decision about a residential location can be regarded as being the outcome of composite speculation within a household and between households. Finally, more than anything else, the problem deals with the interaction between transport and land-use. It is widely accepted that there is a two-way relationship between transport and land-use. Land-use influences travel demands and patterns. The impact of transport on land-use is represented by changes in the level of accessibility, which in turn affects changes in the location of activities. The location of activities has a cyclical relationship with transport because activities generate travel demands and change travel patterns. Transport and land-use keep exchanging mutual responses with each other. Therefore, it is useful to investigate both the impact of transport on land-use and the effect of land-use on transport as a mutual adjustment process.

These three components are regarded in this study as the essential nature of the relationship between transport and the location of activities. There have been much effort and expense over the years in modelling the relationship. No approach, however, satisfactorily represents the relationship in the terms discussed above, which is examined in the literature review chapter. This is the primary motive of this study to develop an alternative model, namely the bid-rent network equilibrium model.

Against this background, the research objectives of this study are:

1. to identify the structural nature of the relationship between transport and the location of activities. This process shows that no existing approach satisfactorily represents the requirements in modelling the relationship.
2. to develop an alternative framework, which is called a bid-rent network equilibrium model, in order to examine the relationship successfully. The discussion is provided by both conceptual and theoretical investigations.
3. to validate and test the bid-rent network equilibrium model to a real network. The purpose of this application is to illustrate the ability of the model in addressing the nature of the relationship that has been identified.

This thesis has six chapters and two appendices. The structure is organised as follows:

In **Chapter One**, a general background of this thesis in terms of the objectives and the structure for the study is presented. In this chapter, the structural nature of the relationship between transport and the location of activities is discussed. Subsequently, a brief overview for the research is provided.

Chapter Two examines the literature to see the extent to which existing models represent the relationship between transport and the location of activities. Models are classified into four categories in terms of their mathematical structure. The ability of each group to address this relationship is examined. Comparative discussion about the advantages and disadvantages of models is provided.

In **Chapter Three**, an alternative framework, called a bid-rent network equilibrium model, is developed. The process shows difficulties in representing the characteristics of locations, namely heterogeneity and indivisibility. A hedonic interpretation is

incorporated to overcome the difficulties. The model addresses the decision-making processes of households. The process is shown as an n -player non-cooperative game. The Nash equilibrium for this game is defined. The game is accompanied by the systematic interaction between transport and land-use. A mutual adjustment process between the two factors represents the interaction. A bi-level mathematical programme is proposed to embody the three components. The resulting formulation is interpreted as an oligopolistic Cournot game, which is an approximation of the n -player non-cooperative game. The model incorporates a multiclass framework, which allows the model to address interclass transport interactions.

In **Chapter Four**, a solution algorithm for the bid-rent network equilibrium model is proposed. Existing algorithms for bi-level mathematical programmes are reviewed. A heuristic routine is suggested which combines a path-based algorithm for solving the lower level problem and the Newton-Raphson procedure for estimating parameters of the upper level model. Simple numerical examples illustrate the operation of the bid-rent network equilibrium model. Results from the solution algorithm are discussed.

In **Chapter Five**, the bid-rent network equilibrium model is applied to a real network. The objective of the application is to demonstrate the ability of the model to investigate the relationship between transport and the location of activities. A medium-sized city is chosen. A base run and two policy runs are simulated. The base run is a simulation conducted using surveyed data. The policy runs represent the introduction of a congestion charge and the release of land for housing development. Class specific behaviour is obtained. The behaviour is demonstrated using network performance indices representing transport impedance and locational attractiveness. Some policy implications of the simulation are presented.

Chapter Six presents the conclusion in terms of the research summary and some suggestions for further work. The contributions of this study are discussed.

The **first Appendix** considers the fundamentals of game theory. This Appendix shows the background framework of the design for the bid-rent network equilibrium model. Game theory is defined and two forms of game representation are presented. Games are classified into cooperative and non-cooperative types. The non-cooperative

games are subdivided into dominant strategy and best response games. The Nash equilibrium is defined in the non-cooperative games under the best response strategy. Applications of game theory to transport studies are examined.

Appendix II represents a general framework of the inverse transform method, which is used in numerical examples of the bid-rent network equilibrium model at Chapter Four. The discussion is provided by both mathematical and graphical illustrations. The illustrations are followed by the description of the generation process for the household data in the numerical examples.

2. Literature Review

2.1. Introduction

The objective of this chapter is to examine the literature to see the extent to which existing models represent the relationship between transport and the location of activities. This examination is a process of comparison and contrast of various models, which is followed by the discussion on the advantages and disadvantages of the models for addressing the interaction. The relationship, traditionally, has been regarded as one of the most important components in land-use and transport interaction studies. This understanding could mean that an investigation into this issue is a core task to appreciate transport and land-use interactions with respect to the representation of the present and the future states of the system.

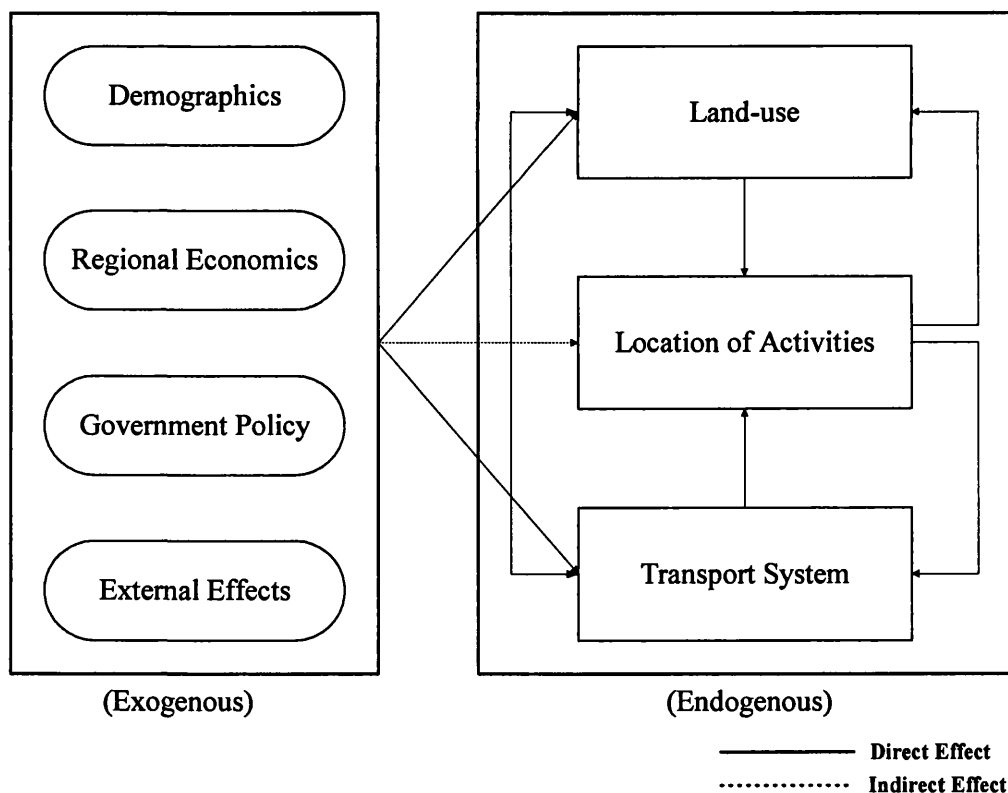


Figure 2-1 A conceptual framework of land-use and transport interaction studies

The land-use transport interaction studies, as shown in Figure 2-1, deal with comprehensive factors. Exogenous factors include demographics, regional economics, government policy, and external effects. Endogenous factors represent the supply-

demand mechanism in the system: the supply involves provision of facilities at a specific location for performing various activities such as housing, manufacturing, and retail *and* provision of transport infrastructures and services between locations; the demand refers to the requirements for services at a specific location as well as travel demands and patterns between locations (Boyce, 1986).

Examining all aspects enumerated is beyond the scope of this review. This study focuses on the decision of an individual regarding the location of activities by the process of the interaction between transport and land-use. This is because much of the important mechanism occurs at this level. As stated in the introductory chapter, four types of locational behaviours have been identified. Three essential components, which represent the unique characteristics of locations, the decision-making process of individuals, and the interaction between transport and land-use, have been proposed as the structural nature of this problem.

There have been many approaches to investigate the relationship. The models are generally very complex to appreciate. This is mainly because the models have been developed for different purposes, and so have been described in rather different terms (Mackett, 1981). Hence, it is difficult to understand exactly what the model structure is. This means that it is impractical to examine whole aspects of each model; instead, the models are considered in terms of a set of criteria that is closely related to the nature of this issue. As for the structural nature, three components have been suggested in this study. Thus, whether a model successfully represents these elements can be a key criterion for evaluating the model. In this context, this study considers the following as a set of criteria with which models are examined:

1. What is the mechanism of a model for the representation of the unique characteristics of locations?
2. How does a model represent the decision-making process of an individual?
3. To what extent does a model represent the interaction between transport and land-use?

As stated above, there are many models that address this problem. It would be almost impossible to enumerate all the models that involve the relationship; instead, it would be useful to investigate models that deal with the interaction directly and explicitly.

For this reason, models whose primary objective is to represent this process are only considered. Therefore, some important models in land-use transport interaction studies are excluded in the literature review. First of all, input-output models such as MEPLAN (Echenique et al., 1990; Hunt and Simmonds, 1993; Mackett, 1991b; Williams, 1994) are not considered. This is because the input-output models mainly represent broad economic changes. Secondly, dynamic simulation models such as MASTER (Mackett, 1988 and 1992) and IRPUD (Wegener, 1986; Wegener et al., 1991) are excluded. This group directly addresses the interaction between transport and the location of activities. The process, however, emphasises the effect of urban dynamics, which is indirect to the purpose of this study. Finally, DELTA (Simmonds and Still, 1998; Simmonds, 1999) is not dealt with in the review, even though the model is currently one of the most operational frameworks in transport and land-use interaction studies. In fact, DELTA belongs to a land-use model. When DELTA analyses the interaction, the model incorporates a transport counterpart.

Even though each model has its own theoretical, methodological, and operational aspects, it is useful to group models into certain categories. In fact, as models have a complex structure, a certain model may not be classified into a particular category. However, it is convenient to impose some form of structure onto the analysis (Mackett, 1981). In this study, models are classified into four categories in terms of their mathematical structure: spatial interaction models, mathematical programming models, random utility models, and bid-rent models. The spatial interaction models represent the flow of trips between locations; the mathematical programming models are designed to produce the optimal allocation of households; the random utility models explain the relationship between transport and the location of activities in terms of the utility-maximising behaviour of decision-makers; the bid-rent models represent the interaction in terms of a bid auction process for occupying locations.

In the next four sections, models are examined using the three criteria. In the subsequent section, comparative discussion on existing models in terms of their advantages and disadvantages is provided. Finally, conclusions are presented.

2.2. Spatial interaction models

2.2.1. Introduction

In this section, spatial interaction models are considered. The modelling process of this group is straightforward. A study area is divided into several zones in an appropriate way. Then, the spatial interaction models represent the number of trips between each pair of locations. The primary purpose of this category of models is to allocate people across an area from a reasonable perspective. This process is represented using a joint function of a measure of transport costs and the activity level of each location. Mathematically, the general function can be expressed as follows:

$$T_{ij} = kR_iE_jf(c_{ij}) \quad (2.1)$$

where T_{ij} is the number of spatial interactions between locations i and j ;

k is a constant;

R_i, E_j are the level of activities in locations i and j respectively; and

$f(c_{ij})$ is a transport cost function between locations i and j .

In this type, the level of spatial interactions is supposed to be proportional to the activity level between locations and inversely proportional to the transport impedance between zones. This reasoning is analogous to Newton's law of gravitational force. Hence, the spatial interaction model is sometimes referred to as a gravity model.

In the next section, the spatial interaction models are reviewed in the context of their historical developments. As mentioned in the previous section, the models are examined in terms of their fitness to the three criteria that have been suggested as the structural nature of the relationship between transport and the location of activities; the indices have meant that the unique characteristics of locations, the systematic decision-making process of households, and the cyclical interaction between transport and land-use. In the subsequent section, the advantages and disadvantages of this group of models are discussed. The discussion concludes that the spatial interaction models would not satisfactorily meet the three components in representing the interaction between transport and the location of activities.

2.2.2. Review

Applications of the spatial interaction model to the relationship between transport and the location of activities trace their roots to the Lowry research of 'A Model of Metropolis' (1964). The Lowry model was an approach to investigating an urban form as represented by a land-use model. The model assumed that, everything else being equal, the place of employment determined the place of residence.

$$P_i' = g \sum_j E_j c_{ij}^{-1} \quad (2.2)$$

where P_i' is residential population in a zone i ;
 g is a population scale factor; and
 E_j is the given number of jobs in a zone j .

The Lowry model was a typical gravity type; the main factors that determined the location of activities were the quantity of economic activities in a zone and transport impedance between locations. The model propagated as follows: the resident population requires 'services', which determines the place of service employment; the service employees require housing; this means that new residents are induced; the additional population requires further services which are fulfilled by additional service employment; the new service employees require housing in relation to their place of work. This round of reasoning continues until no further service employees or households are located (Oryani and Harris, 1996).

The Lowry model was conceptually simple but comprehensive: the model included a variety of locations of activities such as residence, employment, and shopping; the model represented diverse transport components such as trips to work and shopping. For these attributes, the Lowry model has been applied many times. One of the interesting applications was the *Time Oriented Metropolitan Model* (TOMM) (Steger, 1965). TOMM disaggregated households by income level and reformulated the Lowry model incorporating quasi-dynamics. Cripps and Foot (1969) modified the residential location sub-model using an intervening opportunity framework. The modified model was applied to test the impact of a new airport (Cripps and Foot, 1970). The Lowry model was implemented to a new town project (Batty, 1970a) and a sub-regional study (Batty, 1970b). Another example was the *Projective Land-Use*

Model (PLUM) (Goldner et al., 1972), which was proposed to test the effect of incremental growth in employment sections. Later, PLUM was replaced by two models, namely the *Integrated Transportation Land-Use Package* (ITLUP) and the *Projective Optimisation Land-use Information System* (POLIS). ITLUP extended PLUM into a comprehensive land-use and transport interaction model. POLIS reformulated PLUM from a mathematical programming perspective. The former is reviewed in this section, and the latter is considered in the next section.

The Lowry model would not meet the criteria that have been proposed in this study, though the model had reasonable operational contents. First, the model failed to represent the unique characteristics of locations. There was no consideration of the intrinsic attractiveness of locations. Simple proxy variables such as population and the number of jobs addressed the locational attraction. Secondly, the Lowry model offered little investigation into the decision-making process of an individual household. The Lowry model lacked the behavioural interpretation because the model was basically aggregate. The allocation process merely replicated aggregate trends without social or income disaggregation. Furthermore, there was no consideration of the relationship between user classes whose feedback might affect the way land-use was formed. Finally, the model offered limited information on the interaction between transport and land-use. The Lowry model might be used for testing land-use policies by introducing the modification of basic sector land-uses; even in this case, there was no mutual adjustment process between transportation and land-use.

Wilson (1970) generalised the Lowry model incorporating the entropy-maximising principle. The generalisation process can be described as follows:

First, a measure of the relative attractiveness of residential locations was introduced. The attraction measure partly represented locational characteristics.

$$T_{ij} = B_j V_i^\alpha E_j \exp(-\beta c_{ij})$$

$$B_j = \left(\sum_i V_i^\alpha \exp(-\beta c_{ij}) \right)^{-1} \quad (2.3)$$

where T_{ij} is the number of workers who live in a location i and work in j ;

V_i is a measure of the relative attraction of a residential location i ; and

α, β are parameters to be estimated.

Secondly, user groups were disaggregated. The disaggregation considered income groups, wage levels by locations, housing types, and variations in housing prices.

$$T_{ij} = \sum_h \sum_w T_{ij}^{hw} \quad (2.4)$$

where h is a housing type h in a location i and
 w is a level of wage.

Finally, four types of locational behaviour were suggested. This extension, which represents urban dynamics, made the spatial interaction model comprehensive. That is, the model considered different forms of spatial interactions taking into account the effect of changes in transport impedance and locational attraction with lagged time.

$$T_{ij}(t) = \sum_{n=1}^4 T_{ij}^n(t) \quad (2.5)$$

where $n=1$ is locationally unconstrained workers in time t ;
 $n=2$ is fixed residence workers in time t ;
 $n=3$ is fixed workplace workers in time t ; and
 $n=4$ is fixed residence and workplace workers in time t .

To sum up, Wilson's generalisation of the Lowry model made the spatial interaction model flexible; the Wilson model could partly represent the intrinsic locational characteristics; the behavioural difference between user classes could be implemented; the model considered urban dynamics. However, the model could not address the systematic linkage between transport and land-use because the model explained the interaction using given locational benefits and transport costs.

Mackett (1983) noticed the difficulty of the Lowry-Wilson framework in representing the interaction between transport and land-use. The *Leeds Integrated Land-use and Transport (LILT)* model was proposed, which could assess the effect of changes in transport impedance and policy. LILT incorporated the trip distribution and the modal split stages of the traditional four-step travel demand model into the Lowry-Wilson framework. LILT comprehensively represented the relationship between transport costs and the spatial distribution of population, housing, jobs, employment and

shopping. Afterwards, the traffic assignment procedure was incorporated during the research of the *International Study Group on Land-Use Transport Interaction* (ISGLUTI; Webster et al., 1988). The iterative loading capacity-restraint assignment addressed the effect of congestion. The standard Frank-Wolfe (1956) algorithm was used to find an equilibrium travel time. The essence of LILT can be described as follows: households were divided in terms of social status, car ownership groups, and modes of travel; households were allocated to residential and employment areas using a spatial interaction framework; jobs were assigned to three categories, namely primary, secondary, and tertiary; primary jobs were scattered in proportion to the existing distribution; secondary places were allocated on the basis of the previous employment distribution; tertiary sectors responded to the population distribution taking into account the relative cost of travel by each mode. LILT has been rigorously validated and tested (Mackett, 1983; ISGLUTI, Webster et al., 1988). Subsequently, the model was successfully applied to many studies (Mackett, 1990; Mackett, 1991a; Mackett, 1991b; Mackett, 1993a; Mackett, 1993b; Wegener et al., 1991).

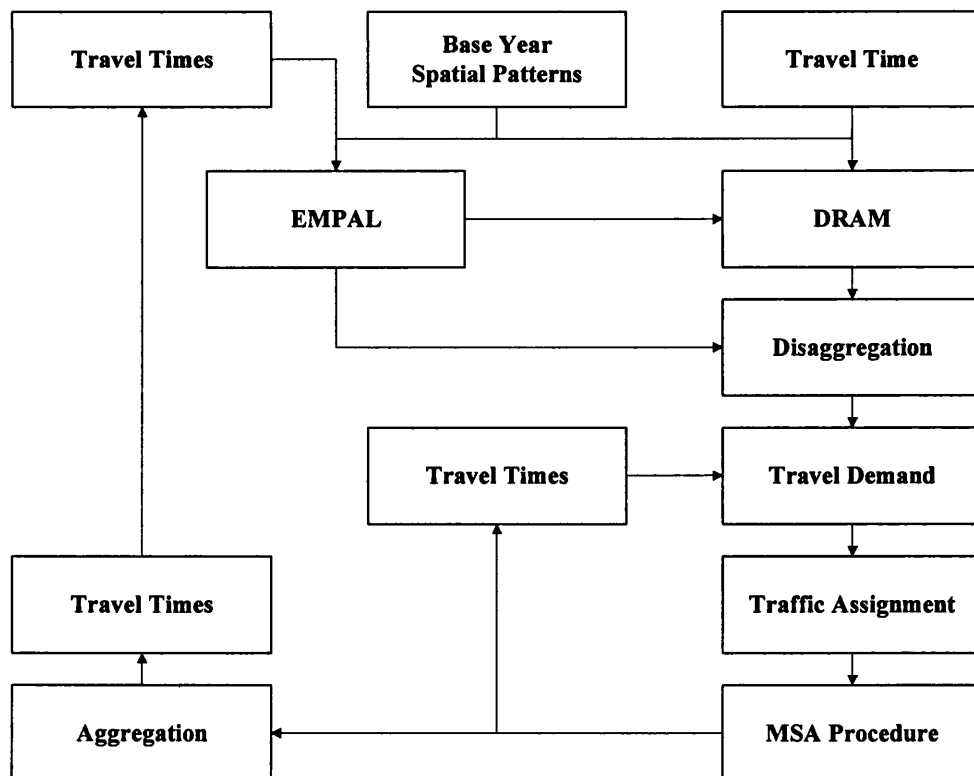


Figure 2-2 The structure of ITLUP, source: Putman Association (1995)

According to Oryani and Harris (1996), the most widely applied spatial interaction model is ITLUP (Putman, 1983; 1984; 1991). The model has been available since the early 1970s and has been improved over the course of the last three decades. ITLUP consists of three main sub-models, namely the *Disaggregated Residential Allocation Model* (DRAM), the *EMPloyment ALlocation model* (EMPAL), and CALIB, which estimates the set of parameters for DRAM and EMPAL.

DRAM is a singly constrained residential allocation model. The model disaggregates households into several groups and forecasts the residential location of households. In this process, a spatial interaction framework is used. This is represented by a function of a measure of zonal attractiveness and accessibility to a workplace where employment locations are defined either outside the model or by the use of EMPAL.

$$N_i^n = \sum_j \left(\sum_k a_{kn} E_j^k \right) \left[\frac{W_i^n f^n(c_{ij})}{\sum_i W_i^n f^n(c_{ij})} \right] \quad (2.6)$$

where N_i^n is the number of type n residents in a zone i ;

a_{kn} is a region-wide coefficient relating the number of type k employees to type n households;

E_j^k is the amount of employment in a sector k in a zone j ;

W_i^n is the composite measure of locational attractiveness in a zone i to employees from a residential group n ; and

$f^n(c_{ij})$ is a transport cost function for type n residents moving from i to j .

The zonal attractiveness is calculated by a multiplicative power function that is similar to the Cobb-Douglas form:

$$W_i^n = (L_i^u)^{q^n} \cdot (1 + x_i)^{r^n} \cdot (L_i^r)^{s^n} \cdot \prod_n \left[1 + \left(\frac{N_i^{n'}}{\sum_n N_i^{n'}} \right) \right]^{b^n} \quad (2.7)$$

where L_i^u is the area of vacant and developable land in a zone i ;

x_i is the proportion of developable land in a zone i which has already been developed;

L_i^r is the area of residential land in a zone i ; and

q^n, r^n, s^n, b^n are parameters to be estimated.

A gamma travel cost function is used for measuring accessibility to a workplace.

$$f^n(c_{ij}) = c_{ij}^{y^n} \exp(-\beta^n c_{ij}) \quad (2.8)$$

where y^n, β^n are parameters to be estimated.

EMPAL forecasts employment locations by types in relation to an attractiveness measure and an employment type with lagged time. The model has a form of a modified singly constrained spatial interaction model.

$$E_{jt}^R = \lambda \left[\sum_i P_{it-1} \cdot A_{it-1}^R \cdot W_{jt-1}^R \cdot f(c_{ijt}) \right] + (1 - \lambda) E_{jt-1}^R \quad (2.9)$$

where E_{jt}^R is the amount of retail employment in a sector R of a zone j in time period t ;

P_{it-1} is the total population of a zone i in prior time period $t-1$;

A_{it-1}^R is a balancing term;

W_{jt-1}^R is the attraction of a zone j for a sector R in prior period $t-1$; and

E_{jt-1}^R is the total employment of a zone j for a sector R in period $t-1$.

The measure of attractiveness is given by a composite index of the form

$$W_{jt-1}^R = (E_{jt-1}^*)^{\alpha^k} L_j^{\beta^k} \quad (2.10)$$

where E_{jt-1}^* is the total employment of a zone j in prior period $t-1$ and

L_j is the total land area of a zone j .

Lastly, CALIB is a calibration programme that estimates the equation coefficients in DRAM and EMPAL. CALIB produces maximum likelihood estimates for the equation coefficients, goodness-of-fit statistics, asymptotic t-tests of the statistical significance of the coefficients, and point elasticity for a sensitivity analysis.

ITLUP has recently been extended into the *METROPolitan Integrated Land-Use System* (METROPILUS). The *METROPILUS* model integrated ITLUP with Arc-

View GIS. The integration enables the model to support an easy-to-use graphical user interface. The system provides increased data analyses and manipulation capabilities as well as a seamless combination with transportation modelling packages such as EMME/2 and TRANPLAN (Lee et al, 1999).

The overall structure of ITLUP is similar to that of LILT. It should be noted that both models have incorporated the traffic assignment stage. As stated above, LILT was associated with iterative loading capacity-restraint assignment that was solved by the Frank-Wolfe (1956) procedure. The traffic assignment in ITLUP showed incremental tree-by-tree loading on the network in which the traffic was added progressively to the links adjusting link travel times. In both models, the stage is integrated as a separate independent model. Thus, the equilibrium procedure between the travel time and the location of residences and workers is not necessarily very sound. Nevertheless, the two models are advanced compared with other spatial interaction models in the sense that both models try to represent the effect of congestion on the transport network. The internalisation of the congestible network would offer a more realistic opportunity to test the effect of changes in transport costs and policies.

2.2.3. Conclusion

This section has reviewed spatial interaction models that are one of the groups of models that investigate the relationship between transport and the location of activities. The spatial interaction models represent the number of trips between each pair of locations using a function involving a measure of transport costs and the level of locational activities. In this process, the models adopt a spatial interaction framework that is an analogous procedure to the law of gravitational force of Newton.

This group originated with Lowry (1964) and has been rigorously applied to many studies because of the dual natural advantages of being both comprehensive and conceptually simple. In addition to these properties, the spatial interaction model has reasonable behavioural contents, which suggests that the model can be used for long-term forecasting. The model is flexible to allow modifications for specific research purposes (Mackett, 1985). The spatial interaction model, however, would not meet the three criteria that have been proposed in this study for representing the nature of the interaction between transport and the location of activities. First of all, the model offers little information on the unique characteristics of locations. Even though the framework could include a measure of locational characteristics, the factor is normally one of the broad aggregate measures such as population and area of zones. Hence, the model would be regarded as representing not so much the intrinsic characteristics of locations but some proxies for locational attractiveness. Secondly, the spatial interaction model is fundamentally aggregate though there could be a disaggregation of user groups. This means that the model crudely represents the decision-making process of an individual household. Finally, the model could be used to test various transport and land-use policies. However, the process of the interaction between transport and land-use would not be explicit because of the lack of a systematic functional relationship between the two components. Furthermore, it is difficult to represent the mutual adjustment process between transport and land-use.

Overall, the limitations which have been discussed above would mean that the spatial interaction model would not be regarded as a suitable framework for representing the relationship between transportation and the location of activities.

2.3. Mathematical programming models

2.3.1. Introduction

Mathematical programming models, which have been applied to the issue of the interaction between transport and the location of activities, are designed to produce an optimal allocation of households across a study area, subject to a set of constraints. The purpose of this type of model is to address the most efficient spatial structure. This objective is represented by minimising costs or maximising benefits rather than describing the spatial distribution process only. In other words, the emphasis shifts from the replication of current situations and the prediction of future outcomes to the design of an efficient spatial structure (Oryani and Harris, 1996).

The basic assumption of the mathematical programming models is that the pattern of the locational distribution of households can be investigated by allocating some quantities that are incorporated into an objective function. The objective function is optimised subject to a set of constraints.

$$\begin{aligned} & \text{Min } Z(\mathbf{x}) \\ & \text{s.t. } g_j(\mathbf{x}) \geq b_j, \quad j = 1, 2, \dots, J \end{aligned} \tag{2.11}$$

where $Z(\cdot)$ is an objective function and $g_j(\cdot)$ are a set of constraints.

This type of model tries to realise some general objectives of urban areas such as accessibility and efficiency (Erlander, 1977). This means that the models seek the best possible urban structure, which has been a long-held aim of transport planners. In this context, this group of models is referred to as normative.

In the next section, a historical review for this group of models is presented. The examination starts with considering early models of this group that have been formulated using linear programming. Then, combined models, which integrate spatial interaction models with network models, are considered. Subsequently, some advantages and disadvantages of this type of model are discussed. The congestible network representation of this group is emphasised as a distinct characteristic.

2.3.2. Review

The early application of the mathematical programming models to the interaction between transport and the location of activities can be found in the Herbert and Stevens study (1960). This research used a linear programming framework and was designed to distribute households to residential areas in an optimal configuration. The Herbert and Stevens model proposed the concept of ‘savings’, which was defined as the difference between ‘total budget’ and ‘costs’, for representing the behaviour of households in the choice of residence; the total budget was defined as the available money of households for consuming residential bundles and other commodities; the costs were defined as the money for consuming other commodities. Reasonable households were assumed to maximise the ‘savings’:

$$\begin{aligned}
 \text{Max } Z &= \sum_{K=1}^U \sum_{i=1}^n \sum_{h=1}^m X_{ih}^K (b_{ih} - c_{ih}^K) \\
 \text{s.t. } \sum_{i=1}^n \sum_{h=1}^m s_{ih} X_{ih}^K &\leq L^K \quad \forall K \\
 \sum_{K=1}^U \sum_{h=1}^m X_{ih}^K &= -N_i \quad \forall i \\
 X_{ih}^K &\leq 0 \quad \forall K, i, h
 \end{aligned} \tag{2.12}$$

- where K is a zone in a study area that is supposed to be homogeneous;
 i is a household group;
 h is a unique combination of the bundle of housing or residential characteristics;
 X_{ih}^K is the number of a household group i consuming the residential bundle h in a zone K ;
 b_{ih} is the residential budget of a household group i ; the budget is used to purchase the residential bundle h ; and
 c_{ih}^K is the annual cost of a household group i consuming the residential bundle h ; the cost is used to buy other commodities.

The Herbert and Stevens model interpreted savings as the ‘rent-paying ability’ of a household group in the sense that the savings were the maximum amount that the household group could pay for the residential bundle. Since the model assumed households were reasonable, the resulting residential distribution would signify the

Pareto optimal; no household group could increase the savings without reducing the savings of other groups and the total savings of the area.

The Herbert-Stevens model had an attractive framework to represent the behaviour of households from a theoretical perspective and the model provided an analogous solution to Alonso's (1965) the unique bid-rent theory. The nature of the model, however, would not meet the criteria that have been adopted in this study. First, there was a crude representation on the locational characteristics. The model supposed that there were several zones in an area; however, there was no systematic examination in the intrinsic difference between the zones. Furthermore, the mechanism in terms of how the locational characteristics affect the location of activities and travel patterns was not investigated. Secondly, the model had limits to address the decision-making process of an individual household. The model could incorporate social or income-group disaggregation. The classification, however, was rather crude because the model was fundamentally aggregate. Finally, the model had no interaction mechanism between transport and land-use. In addition to these weaknesses, the model required such vast quantities of data, which made the model inoperational.

TOPAZ (*Technique for the Optimal Placement of Activities in Zones*) was developed to be a general planning tool (Brotchie, 1969). The model has been improved over the course of practical applications (Sharpe and Brotchie, 1972; Brotchie et al, 1980).

TOPAZ allocated activities to locations on the basis of maximising the net benefit that was assumed to be obtained from spatial interactions and land-use:

$$\begin{aligned}
 \text{Max } \sum_r \sum_t U(X_{rit}) &= \sum_{ri} \sum_{sj} \sum_m \sum_t T_{risjmt} \cdot d_{ijmt} \cdot x'_{risjmt} + \sum_{ri} \sum_t X_{rit} \cdot x_{rit}^E \\
 \text{s.t. } \sum_r \sum_t \left(\frac{X_{rit}}{j_r} \right) &\leq L_i \\
 \sum_i X_{rit} &= X_{rt} \quad \forall \quad r, t \\
 X_{rit} &\geq 0 \quad \forall \quad r, i, t
 \end{aligned} \tag{2.13}$$

where X_{rt} is the total activities of a type r to be loaded in time period t ;

X_{rit} is the portion of X_{rt} to be allocated to a zone i during period t ;

$U(X_{rit})$ is the total merit of the portion X_{rit} ;

- T_{risjmt} is the number of interactions between the activity r of a zone i and the activity s of a zone j with a travel mode m in period t ;
- d_{ijmt} is the distance between i and j with a travel mode m in period t ;
- x_{risjmt}^I is the unit of the net interaction benefit;
- x_{rit}^E is the unit of the net establishing and operating benefit;
- j_r is the density per a unit area of an activity r ; and
- L_i is an area available for all activities in a zone i .

TOPAZ represented transport components in terms of interactions between activities, but the mechanism was rather crude. Later, TOPAZ reformulated the transport sub model using Wilson's (1970) entropy maximisation (Webster et al., 1988, p. 497). The sub model defined interactions as the number of trips between activity locations:

$$T_{risjmt} = A_{rit} \cdot B_{sjt} \cdot O_{rit} \cdot D_{sjt} \cdot f(x_{risjmt}^I) \quad (2.14)$$

where O_{ri} the number of trips generated from i by an activity r in t ;

D_{sj} the number of trips attracted to j by an activity s in t ; and

A_{rit}, B_{sjt} are flow balancing factors.

The overall structure of TOPAZ was similar to that of the Herbert and Stevens model (1960); both models used a linear programming framework with a maximisation perspective. However, it is important to note that TOPAZ might represent the mutual adjustment process between transport and land-use. By introducing the transport sub model, the model structure of TOPAZ was similar to a bi-level programme in the sense that the main objective function had the transport sub model in the lower level. Therefore, if TOPAZ had incorporated the iterative process, the model could have produced the endogenously determined number of interactions.

The Projective Optimisation Land-use Information System (POLIS) was developed to provide a forecast of land-use, employment, housing, population, and transport demand (Prastacos, 1985; Prastacos, 1986a; Prastacos, 1986b). As noted in the review of the spatial interaction models, POLIS was one of the substitutes of PLUM. The model was formulated in a nonlinear programming framework. The model sought to

represent a simultaneous decision-making procedure rather than the sequential process that can be shown in Lowry derivative models.

POLIS incorporated Williams' (1977) surplus formula into the Wilson model (1970) from a mathematical programming perspective. The model assumed that the residential choice of households would be affected by a workplace location, a travel mode to work, and shopping behaviour:

$$\begin{aligned}
 \text{Max } Z(T_{ijm}, S_{ij}) = & \left[-\frac{1}{\beta^w} \sum_{ijm} T_{ijm} \left\{ \ln \left(\frac{1}{W_i} \sum_m T_{ijm} \right) - 1 \right\} \right] \\
 & + \left[-\frac{1}{\lambda} \sum_{ijm} T_{ijm} \{ \ln T_{ijm} - 1 \} - \sum_{ijm} T_{ijm} c_{ijm}^w \right] \\
 & + \left[-\frac{1}{\beta^s} \sum_{ij} S_{ij} \left\{ \ln \left(\frac{S_{ij}}{W_j^s} \right) - 1 \right\} - \sum_{ij} S_{ij} c_{ij}^s \right] \\
 \text{s.t. } W_i = & \exp(\beta^w u_i^w) \\
 \sum_{jm} T_{ijm} - d \sum_j S_{ij} = & 0 \\
 \sum_{im} T_{ijm} - d' \sum_i S_{ij} - d'' E_j^B = & 0 \\
 T_{ijm}, S_{ij} \geq & 0
 \end{aligned} \tag{2.15}$$

where T_{ijm}, S_{ij} are the surplus functions of work and shopping trips respectively;

u_i^w is the utility of living at a location i ;

E_j^B is the basic employment in a zone j ;

d, d', d'' are scale parameters; and

$\beta^w, \beta^s, \lambda$ are parameters that convert the utility of trip-makings into monetary units compatible with the transport cost incurred.

In the formulation, the objective function represented the sum of three surplus functions. This framework could flexibly incorporate other components. This would mean that the model could represent a multidimensional decision-making process.

Even though POLIS had the complex mathematical structure, the model produced, in the end, a doubly constrained spatial interaction model that was equivalent to the Wilson model (1970):

$$\begin{aligned}
T_{ijm} &= A_i^w H_i B_j^w E_j \exp(-\beta' \tilde{c}_{ij}) \frac{\exp(-\lambda c_{ijm}^w)}{\sum_m \exp(-\lambda c_{ijm}^w)} \\
S_{ij} &= A_i^s H_i B_j^s E_j \exp(-\beta^s c_{ij}^s) \\
A_i^w &= \left[\sum_j B_j^w E_j \exp(-\beta' \tilde{c}_{ij}) \right]^{-1}, \quad B_j^w = \left[\sum_i A_i^w H_i \exp(-\beta' \tilde{c}_{ij}) \right]^{-1} \\
A_i^s &= \left[\sum_j B_j^s E_j \exp(-\beta^s c_{ij}^s) \right]^{-1}, \quad B_j^s = \left[\sum_i A_i^s H_i \exp(-\beta^s c_{ij}^s) \right]^{-1}
\end{aligned} \tag{2.16}$$

where E_j^s is the total service employment in a zone j .

There was no significant progress in POLIS to represent the three criteria compared with the existing models. Like the Herbert and Stevens model or TOPAZ, the model offered little information on the unique characteristics of locations; POLIS lacked the behavioural interpretation of decision-makers because the model was aggregate; the model addressed the crude interaction between transport and land-use.

Important progress in terms of the interaction between transport and land-use has been made in combined models. This group of models claimed that both the spatial interaction and mathematical programming models could not successfully represent the mutual adjustment process between transport and land-use because the two groups assumed fixed transport costs. The combined model integrated a network model with a spatial interaction framework tackling the crude representation (Evans, 1973; Evans, 1976; Florian et al., 1975; Erlander, 1977; Erlander et al., 1979; Boyce and Southworth, 1979; Los, 1979; Boyce, 1980).

$$\begin{aligned}
\text{Min } Z &= \sum_a \int_0^{t_a} t_a(x) dx + \frac{1}{\beta} \sum_i \sum_j T_{ij} \ln T_{ij} \\
\text{s.t. } \sum_k f_{ij}^k &= T_{ij} & \forall i, j \\
\sum_j T_{ij} &= O_i & \forall i \\
\sum_i T_{ij} &= D_j & \forall j \\
f_{ij}^k &\geq 0 & \forall i, j, k
\end{aligned} \tag{2.17}$$

The above formulation produced the following:

$$T_{ij} = T \frac{V_i^\alpha W_j^\beta \exp(-\beta c_{ij})}{\sum_i \sum_j V_i^\alpha W_j^\beta \exp(-\beta c_{ij})} \quad (2.18)$$

$$\begin{aligned} \text{if } f_{ij}^k > 0, \text{ then } \sum_a t_a(v_a) \delta_{ij}^{a,k} &= c_{ij} \\ \text{if } f_{ij}^k = 0, \text{ then } \sum_a t_a(v_a) \delta_{ij}^{a,k} &\geq c_{ij} \end{aligned} \quad (1.19)$$

where W_j^β is a measure of the relative attractiveness in an employment location j . Equation (2.18) satisfies the constraints of trip ends, spatial interactions, and zonal attractiveness; equation (2.19) is needed to satisfy the Wardrop principle.

It should be noted that transport costs in the combined model is determined endogenously. The endogenous determination was achieved by the interaction between transport and land-use. This means that the model can consider a congestible transport network, and so the endogenous transport impedance is regarded as an innovative contribution of the combined model. Another important point that should be noted in the formulation is that the combined model can incorporate the disaggregation and urban dynamics processes of the Wilson (1970) model. The processes, however, are not specified in the formulation for the concise discussion.

Kim (1979, 1983) proposed a model that had a different perspective but was related to the group of the combined models, namely a general equilibrium model between the demand for and supply of transport and activity locations. The model could produce the endogenously determined zonal travel demands, link congestion costs, and the optimal amount of production in efficient densities of land-use.

$$\text{Min } Z = \sum_a \int_0^a t_a(x) dx + \sum_i \sum_r d_i^r(E_i^r) \quad (2.20)$$

where E_i^r is the unit of a commodity r exported from i and

$d_i^r(\cdot)$ is the unit of costs of an exporting a commodity r from a zone i .

Sheffi (1985, p. 166) suggested an alternative formulation that highlighted an explicit representation of a measure of locational attractiveness:

$$\text{Min } Z(\mathbf{v}, \mathbf{T}) = \sum_a \int_0^a t_a(x) dx - \sum_i \sum_j M_j T_{ij} \quad (2.21)$$

where M_j is a measure of locational attractiveness. The attractiveness measure could be either fixed aggregate indices such as the number of houses or some form of demand functions. When the formulation incorporates a demand function in the form of a gravity model generalised by the Entropy maximisation (Wilson, 1970), the structure is identical to that of Evans (1973; 1976) shown in equation (2.17).

Chu (1990) noted that the combined models could not satisfactorily represent the locational behaviour in which households had a different level of 'captivity'. A typical example of the different captivity is that the observed interaction patterns are represented by both compulsory and discretionary behaviours. Compulsory interactions are those made even in the worst conditions. In contrast, discretionary interactions are less regular both in time and space. Therefore, the two behaviours would require different modelling approaches. The Dogit distribution model (Gaudry and Wills, 1979) was integrated to a network model in order to represent the captivity:

$$T_{ij} = T \cdot \left(\frac{\sigma_{ij}}{1 + \sum_j \sigma_{ij}} + \frac{1}{1 + \sum_j \sigma_{ij}} \cdot \frac{\exp(M_j - \beta c_{ij})}{\sum_j \exp(M_j - \beta c_{ij})} \right)$$

$$\text{if } f_{ij}^k > 0, \quad \text{then } \sum_a t_a(v_a) \delta_{ij}^{a,k} = c_{ij} \quad (2.22)$$

$$\text{if } f_{ij}^k = 0, \quad \text{then } \sum_a t_a(v_a) \delta_{ij}^{a,k} \geq c_{ij}$$

where σ_{ij} is a nonnegative parameter that represents the captivity odds of households in an origin i whose destination is j .

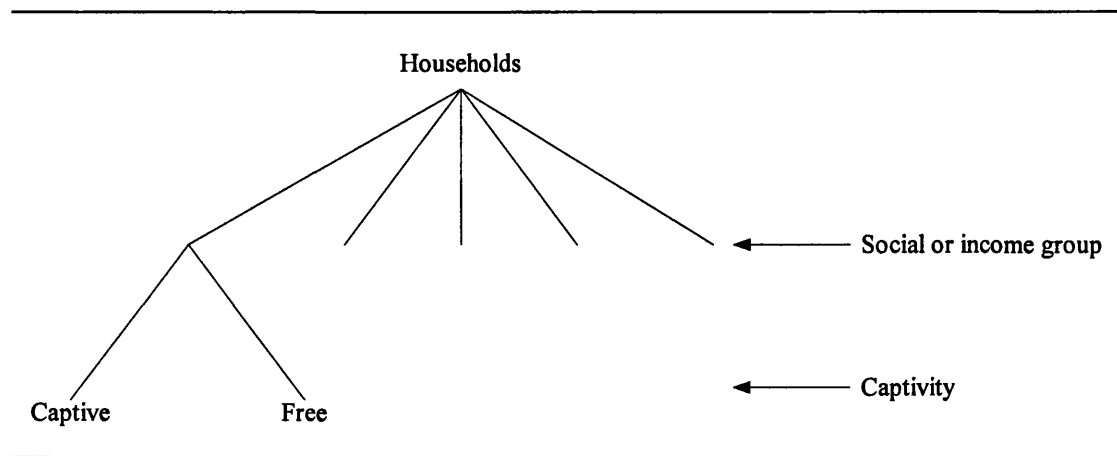


Figure 2-3 Two-stage disaggregation in the Chu model

The Chu model has a two-stage disaggregation of user groups to represent the different captivity. Figure 2-3 shows a conceptual diagram for the disaggregation process. In the diagram, the model introduced the second stage disaggregation while conventional models normally suggested the first stage only. This complexity results from the aggregate nature of the mathematical programming models. In other words, the aggregate model requires an extra classification of user groups in order to describe the various behaviours of decision-makers. The behaviour, in the end, could be successfully represented by incorporating a disaggregate modelling framework.

A multiclass perspective has been incorporated in the combined model (Lam and Huang, 1992a; Lam and Huang, 1992b; Nagurney and Dong, 2002b). Conventional combined models implicitly assumed that there was only one homogenous user group when the models calculated transport costs even though the approaches disaggregated households into several classes. Furthermore, the interaction between classes was merely implemented by exogenously determined user specific parameters. The multiclass framework tackled this crude representation by formulating an explicit interaction between household classes. In this framework, a class was defined as a group of households that perceived the criteria associated with transport components in an identical fashion. In particular, this perspective could represent multimode problems that were unlikely to be considered in the conventional combined models. Clearly, the multiclass model assumed that the transport impedance of a class depended on the flow of its own class as well as on that of other classes. The interaction could be asymmetric. In this case, there is no equivalent convex mathematical programming formulation for representing the multiclass interaction. Instead, Lam and Huang (1992a) incorporated Van Vliet and others' (1986) normalising approach into the existing combined model:

$$\text{Min } Z = \sum_a \int_0^{v_a} t_a(x) dx + \sum_a \sum_k \sum_m \frac{b_k^m}{b_o^m} \cdot d_a^k \cdot v_a^m + \frac{1}{\beta^m} \sum_i \sum_j \sum_m T_{ij}^m \ln T_{ij}^m \quad (2.23)$$

where d_a^k is the k^{th} fixed cost component on a link a and

b_o^m, b_k^m are parameters specific to a class m .

Nagurney and Dong (2002b) formulated the same problem as Lam and Huang (1992a) using a variational inequality. The variational inequality formulation is more

general than the normalising counterpart in the sense that the formulation can handle the asymmetric interaction between user classes (Smith, 1979; Dafermos, 1980).

$$\sum_m \sum_a g_a^m(\mathbf{v}^*) (v_a^m - v_a^{m*}) \geq 0 \quad (2.24)$$

where $g_a^m(\cdot)$ is a generalised cost function on a link a for a class m ;

v_a^{m*} is the optimal volume of a class m on a link a ; and

\mathbf{v}^* is the optimal link volume vector where $\mathbf{v} = \{v_1^1, \dots, v_a^1; \dots; v_1^m, \dots, v_a^m\}$.

Another extension of the combined models can be found in the study of the incorporation of endogenous locational costs that are necessary for performing activities (Oppenheim, 1993). Oppenheim argued that zonal attractiveness would be a function of trip ends; the trip ends themselves would be a function of locational costs. Congestion is a typical example of the locational costs. This is because congestion at a specific location may create additional travel time. An alternative formulation was proposed in which decision-makers were assumed to choose locations so as to minimise the sum of stochastic locational costs and deterministic travel costs.

$$\text{Min } Z(\mathbf{v}, \mathbf{T}) = \sum_a \int_0^{t_a} t_a(x) dx + \frac{1}{\beta} \sum_i \sum_j T_{ij} \ln T_{ij} + \sum_j \left(a_j D_j + \int_0^{D_j} d_j(x) dx \right) \quad (2.25)$$

where, $d_j(\cdot)$ is a destination cost function.

The Oppenheim model was originally developed for travel demand analyses, but the model suggested a useful interpretation for representing the relationship between transportation and the location of activities. The locational cost in the Oppenheim model, which is determined endogenously, could be directly related to locational opportunity costs. This might mean that the model could produce the endogenously calculated locational opportunity cost as well as the transport cost, though the interaction process between the two factors was not explicit.

2.3.3. Conclusion

This section has reviewed mathematical programming models that have been applied to the research for the interaction between transport and the location of activities. The models represent an optimal allocation of households across an area. Hence, the primary purpose of this group is to suggest an efficient urban structure. This means that the models are linked to find system efficiency such as cost minimisation or benefit maximisation. Early models of this type were formulated using linear or non-linear optimisation programming; later, combined models were developed. The combined models integrated a spatial interaction framework with a network model, which highlighted the interaction between transport and land-use.

This type of model has a simple mathematical form and does not necessarily account for a wide range of empirical properties for a study area. However, the nature of this group would not satisfactorily meet the criteria that have been adopted for evaluating models in this study. First of all, the models have little consideration for the unique characteristics of locations. Like the spatial interaction models, the mathematical programming models use broad aggregate measures such as population or the area of zones. This means that the models crudely represent locational features. Secondly, the models fail to represent the decision-making process of an individual household. Although most models in this group disaggregate population into social or income groups, the nature of the aggregate model structure means difficulty in addressing the behaviour of an individual household. Finally, the mathematical programming models show the partial interaction between transport and land-use. The greatest advantage of this type would be the representation of interaction between transport and land-use. In particular, the combined models produce endogenously determined transport costs that are generated by the mutual adjustment process between land-use and transport. The representation, however, is partial in the sense that the locational attractiveness is assumed to be fixed and the interaction is initiated by transport components only. In spite of the partial mechanism, the endogenous transport cost means that the model addresses a congestible transport network. This is a distinct contribution of the combined model to the studies of land-use transport interactions.

2.4. Random utility models

2.4.1. Introduction

Random utility models address the relationship between transport and the location of activities in terms of the utility-maximising behaviour of a household. This group has been one of the most frequently used frameworks for investigating the relationship.

The random utility models functionalise the connection between the characteristics of locations and the behaviour of decision-makers. Let \mathbb{C} be a feasible choice set for a household h . It is assumed that the choice set of each household is well specified from a scientific perspective. A household has a utility function in relation to the attributes of a location and the characteristics of a household. Not all attributes of a location and a household are observed. Hence, a random term is introduced to consider the unobserved attributes. As a result, the total utility of any alternative location $i \in \mathbb{C}$ is expressed by the sum of the observed and unobserved components.

$$U(i) = V_i + \varepsilon_i \quad (2.26)$$

where V_i is the system component of an alternative location i and
 ε_i is the random component of an alternative location i .

The probability that any alternative location i is chosen by a household is given by

$$\begin{aligned} P(i) &= \text{prob}[U_i \geq U_{i'}, \forall i' \in \mathbb{C}] \\ &= \text{prob}[V_i + \varepsilon_i \geq \text{Max}(V_{i'} + \varepsilon_{i'}), \forall i' \in \mathbb{C}, i' \neq i] \end{aligned} \quad (2.27)$$

Historical developments of the random utility models for representing the relationship between transport and the location of activities are reviewed in the next section. The review is considered in terms of theoretical investigations into model formulations and practical applications to diverse transportation problems. In the following section, the advantages and disadvantages of this group of models are discussed. The discussion concludes that the random utility model would not be an appropriate framework for representing the interaction between transport and the location of activities even though this group has some attractive features.

2.4.2. Review

In most applications of the random utility model, feasible alternatives have a combination of underlying choice dimensions, namely multiple dimensions. Unlike a single dimensional problem, the multidimensional cases have difficulty in defining a feasible choice set because some elements in the multidimensional choice set are logically related. For example, in the choice problem of residence and a travel mode, a particular location may not have a transit service. This suggests that there is a linkage between the particular location and the private car. Two theoretical approaches have been investigated to represent the multidimensional problems, namely joint and nested logit models. A joint logit model assumes that an individual simultaneously makes a decision concerning the multidimensional problem. In this framework, alternatives are supposed to have common observed elements specific to each dimension. This suggests that the model does not consider unobserved components specific to each dimension. In contrast, a nested logit model assumes that a decision is sequentially made incorporating both observed and unobserved components; one of the unobserved components is supposed to be negligible. A nested logit model is more flexible than a joint logit model in the sense that the model opens the possibility of having unobserved components specific to each dimension.

An early application of a joint logit model to represent the interaction between transport and the location of activities can be found in the rental residence study of Quigley (1976). The model proposed the total 18 alternatives that were a joint set of a dwelling type d and a location with characteristics vector \mathbf{z} . The utility function was expressed by the sum of system and probabilistic elements.

$$U(d, \mathbf{z}) = V_d + V_{\mathbf{z}} + V_{d\mathbf{z}} + \varepsilon_{d\mathbf{z}}, \forall (d, \mathbf{z}) \in \mathbb{C} \quad (2.28)$$

where V_d is the common system components of a dwelling type d ;

$V_{\mathbf{z}}$ is the common system components of a location with the vector of housing characteristics \mathbf{z} ;

$V_{d\mathbf{z}}$ is the remaining system components specific to the combination of a dwelling and a location (d, \mathbf{z}) ; and

$\varepsilon_{d\mathbf{z}}$ a random utility component.

The Quigley model was expressed as a multinomial logit type by assuming the disturbance term ε_{dz} distributed IID Gumbel.

$$P(d, z) = \frac{\exp(V_d + V_z + V_{dz})}{\sum_{(d', z')} \exp(V_{d'} + V_{z'} + V_{dz'})}, \forall (d', z') \in \mathbb{C} \quad (2.29)$$

It would be useful to reformulate a joint logit model in terms of marginal and conditional probabilities for the subsequent discussion. When a choice problem is expressed in terms of a conditional probability, the formulation implicitly assumes that the decision-making process of a household is sequential. For example, in the study of Quigley (1976), say an individual household is supposed to select a dwelling type d first; subsequently, a residential location is chosen given the dwelling type. Then, the probability of the composite alternative (d, z) being chosen is given:

$$P(d, z) = P(d) \cdot P(z | d) \quad (2.30)$$

The marginal probability $P(d)$ is calculated by summing each joint probability.

$$\begin{aligned} P(d) &= \sum_z P(d, z), \forall z \in \mathbb{C}_z \\ &= \sum_z \frac{\exp(V_d + V_z + V_{dz})}{\sum_{(d', z')} \exp(V_{d'} + V_{z'} + V_{dz'})} \end{aligned} \quad (2.31)$$

where \mathbb{C}_z is the restricted one dimensional choice set of a location.

The probability $P(d)$ is rewritten as a multinomial logit model adding the term V'_d .

$$P(d) = \frac{\exp(V_d + V'_d)}{\sum_{d'} \exp(V_{d'} + V'_{d'})}, \forall d \in \mathbb{C}_d \quad (2.32)$$

where $V'_d = \ln \sum_z \exp(V_z + V_{dz})$ and

\mathbb{C}_d is the restricted dimensional choice set of a dwelling type.

The conditional probability $P(z | d)$ is calculated combining equations (2.29) with (2.32). The conditional probability is also expressed by a multinomial logit type with the restricted dimensional choice set of a location.

$$\begin{aligned}
P(\mathbf{z} | d) &= P(d, \mathbf{z}) / P(d) \\
&= \frac{\exp(V_{\mathbf{z}} + V_{d\mathbf{z}})}{\sum_{\mathbf{z}'} \exp(V_{\mathbf{z}'} + V_{d\mathbf{z}'})}, \forall \mathbf{z} \in \mathbb{C}_{\mathbf{z}}
\end{aligned} \tag{2.33}$$

Finally, the probability of the composite alternative (\mathbf{z}, d) being chosen can be expressed by combining the marginal and conditional probabilities.

$$P(\mathbf{z}, d) = P(d) \cdot P(\mathbf{z} | d) = \frac{\exp(V_d + V_d')}{\sum_{d'} \exp(V_{d'} + V_{d'})} \cdot \frac{\exp(V_{\mathbf{z}} + V_{d\mathbf{z}})}{\sum_{\mathbf{z}'} \exp(V_{\mathbf{z}'} + V_{d\mathbf{z}'})} \tag{2.34}$$

The formulation in equation (2.34) shows a sequential decision-making process. This means that a joint logit model can be available to represent a simultaneous as well as a sequential decision-making process provided with some modifications. A joint logit model, however, is no longer appropriate when an alternative has unobserved components specific to each dimension.

When a choice set has unobserved as well as observed elements specific to each dimension, a utility function should be extended as follows:

$$U(d, \mathbf{z}) = V_d + V_{\mathbf{z}} + V_{d\mathbf{z}} + \varepsilon_d + \varepsilon_{\mathbf{z}} + \varepsilon_{d\mathbf{z}} \tag{2.35}$$

where ε_d is the unobserved components of a dwelling unit;

$\varepsilon_{\mathbf{z}}$ is the unobserved components of a location with characteristics \mathbf{z} ; and

$\varepsilon_{d\mathbf{z}}$ is the remaining unobserved components.

In particular, a joint logit model is no longer applicable to the situation in which some alternatives are correlated. This is because alternatives share common unobserved components within the dimension. In this case, the covariance of utility is completely dependent on the shared components. A joint logit model cannot represent this relationship. For example, in the Quigley model, all other components except a location \mathbf{z} have no influence on the determination of the covariance of utility.

$$\begin{aligned}
&\text{cov}[U(d, \mathbf{z}), U(d', \mathbf{z})] \\
&= \text{cov}(\varepsilon_d + \varepsilon_{\mathbf{z}} + \varepsilon_{d\mathbf{z}}, \varepsilon_{d'} + \varepsilon_{\mathbf{z}} + \varepsilon_{d'\mathbf{z}}) \\
&= \text{var}(\varepsilon_{\mathbf{z}})
\end{aligned} \tag{2.36}$$

A nested logit model is more general and flexible than a joint logit model. A nested logit model assumes that one of the dimensions has correlated components but the unobserved components of the dimension are negligible. For example, in the Quigley model (1976), if residential locations are supposed to have some common components and the unobserved components of locations are negligible, namely $\varepsilon_z = 0$, the utility function is reformulated as follows:

$$U(d, \mathbf{z}) = V_d + V_z + V_{dz} + \varepsilon_d + \varepsilon_{dz} \quad (2.37)$$

The marginal probability for the dwelling type d is calculated if and only if an alternative containing d has the highest utility.

$$P(d) = \text{prob} \left[\max_{\mathbf{z}} U(d, \mathbf{z}) \geq \max_{\mathbf{z}} U(d', \mathbf{z}), \forall d' \in \mathbb{C}_d, d' \neq d \right] \quad (2.38)$$

where the term $\max_{\mathbf{z}} U(d, \mathbf{z})$ is the utility of the best alternative in \mathbb{C}_d .

The formulation in equation (2.38) means that the dwelling type d is chosen if the best alternative with d is better than without d . It is noted that a choice set is restricted into the single dimension, namely from \mathbb{C} to \mathbb{C}_d .

Since $U(d, \mathbf{z}) = V_d + V_z + V_{dz} + \varepsilon_d + \varepsilon_{dz}$, equation (2.38) is rewritten as follows:

$$\text{prob} \left[V_d + \varepsilon_d + \max_{\mathbf{z}} (V_z + V_{dz} + \varepsilon_{dz}) \geq V_{d'} + \varepsilon_{d'} + \max_{\mathbf{z}} (V_z + V_{dz} + \varepsilon_{dz}) \right] \quad (2.39)$$

where $\forall d' \in \mathbb{C}_d, d' \neq d$.

The term ε_{dz} is assumed to be distributed as IID Gumbel with parameter μ_z , then

$$E \left[\max_{\mathbf{z}} (V_z + V_{dz} + \varepsilon_{dz}) \right] = \frac{1}{\mu_z} \ln \sum_{\mathbf{z}} \exp \mu_z (V_z + V_{dz}) = V'_d \quad (2.40)$$

The combination of equation (2.39) and (2.40) provides

$$P(d) = \text{prob} \left[V_d + \varepsilon_d + V'_d + \varepsilon'_d \geq V_{d'} + \varepsilon_{d'} + V'_{d'} + \varepsilon'_{d'}, d' \in \mathbb{C}_d, d' \neq d \right] \quad (2.41)$$

Equation (2.41) is expressed as a multinomial logit model if the combined disturbance $\varepsilon_d + \varepsilon'_d$ is assumed to be distributed as IID Gumbel with parameter μ_d .

$$P(d) = \frac{\exp \mu_d (V_d + V'_d)}{\sum_{d'} \exp \mu_d (V_{d'} + V'_{d'})} \quad (2.42)$$

One of the differences between joint and nested logit models is the values of the parameters μ_d and μ_z . A joint logit model assumes that both values are equal to one. In contrast, a nested logit model is flexible in the values.

The conditional probability $P(z|d)$ is derived from a similar way to the case of the marginal probability.

$$\begin{aligned} P(z|d) &= \text{prob}[U(z, d) \geq U(z', d), \forall z' \in \mathbb{C}_z, z' \neq z | d] \\ &= \text{prob}[V_d + V_z + V_{dz} + \varepsilon_d + \varepsilon_{dz} \geq V_d + V_{z'} + V_{dz'} + \varepsilon_d + \varepsilon_{dz'}] \\ &= \text{prob}[V_z + V_{dz} + \varepsilon_{dz} \geq V_{z'} + V_{dz'} + \varepsilon_{dz'}, \forall z' \in \mathbb{C}_z, z' \neq z | d] \end{aligned} \quad (2.43)$$

The random term ε_{dz} is supposed to follow IID Gumbel distribution, and then the equation (2.43) is expressed as a multinomial logit model.

$$P(z|d) = \frac{\exp \mu_z (V_z + V_{dz})}{\sum_{z'} \exp \mu_z (V_{z'} + V_{dz'})} \quad (2.44)$$

A nested logit model has two scale parameters, namely μ_d and μ_z . In practice, the parameters are identified using observed data. Since the value of the scale parameters is expressed by the ratio, one of the parameters can be set as one without any affect on the result (Ben-Akiva and Lerman, 1987). In this section, the parameter μ_z is set as one for the comparison with a joint logit model.

Finally, the probability of the composite alternative (z, d) being chosen can be expressed combining the marginal with the conditional probabilities.

$$P(z, d) = P(d) \cdot P(z|d) = \frac{\exp \mu_d (V_d + V'_d)}{\sum_{d'} \exp \mu_d (V_{d'} + V'_{d'})} \cdot \frac{\exp (V_z + V_{dz})}{\sum_{z'} \exp (V_{z'} + V_{dz'})} \quad (2.45)$$

where $V'_d = \ln \sum_z \exp (V_z + V_{dz})$.

The functional form of the systematic components is an important point that should be noted. In most cases, a form of a linear-in-parameter function is adopted for computational convenience.

$$V_d = \hat{\beta}_d^T \mathbf{x}_d; \quad V_z = \beta_z^T \mathbf{x}_z; \quad V_{dz} = \beta_{dz}^T \mathbf{x}_{dz} \quad (2.46)$$

where \mathbf{x}_d , \mathbf{x}_z , and \mathbf{x}_{dz} correspond to the attributes of dwelling types, locations, and combinations of dwelling types and locations, respectively, and $\hat{\beta}_d^T$, β_z^T , and β_{dz}^T are the vectors of corresponding parameters.

$\hat{\beta}_d^T$ is changed into β_d^T and μ_d into σ for notational consistency with the preceding research, and then the equation (2.45) is rewritten as follows:

$$\begin{aligned} P(\mathbf{z}, d) &= P(d) \cdot P(\mathbf{z} | d) \\ &= \frac{\exp(\beta_d^T \mathbf{x}_d + \sigma V'_d)}{\sum_{d'} \exp(\beta_{d'}^T \mathbf{x}_{d'} + \sigma V'_{d'})} \cdot \frac{\exp(\beta_z^T \mathbf{x}_z + \beta_{dz}^T \mathbf{x}_{dz})}{\sum_{z'} \exp(\beta_{z'}^T \mathbf{x}_{z'} + \beta_{dz'}^T \mathbf{x}_{dz'})} \end{aligned} \quad (2.47)$$

where $V'_d = \ln \sum_{\mathbf{z}} \exp(\beta_z^T \mathbf{x}_z + \beta_{dz}^T \mathbf{x}_{dz})$.

The term V'_d represents the utility of the best alternative in the subset; hence, the factor indicates the expected maximum utility of a sub set of alternatives to an individual. In this context, the term has been interpreted as a measure of ‘inclusive value’ (McFadden, 1978) or ‘accessibility’ (Ben-Akiva and Lerman, 1979). The value of σ is imposed between zero and one. McFadden (1978) interpreted σ as an index for the degree of independence of alternatives. Clearly, a nested logit model is equivalent to a multinomial logit model if the parameter σ is set as one.

Lerman (1977) implemented both joint and nested logit models with a sample of 177 skilled single-worker households in Washington DC, USA. The study argued that the locational decision of a household is closely related to other choices of housing, car ownership, and travel mode to work. A four dimensional choice set called ‘mobility bundles’ was proposed and two types of model were estimated, namely constrained and unconstrained models. In the constrained model, a joint logit type, a household was supposed to simultaneously choose the four-dimensional alternative. In the unconstrained model, a nested logit type, a household was assumed to choose three-

dimensional bundles of housing, car ownership, and travel mode first; subsequently, a location was selected supposing locations had shared components. This meant that the choice structure was designed by a sequential process. The result, $\sigma = 0.492$, showed that there was a substantial correlation among the unobserved components within locations. The correlation resulted from the use of a census tract that was a group of housing units. Therefore, other conditions being equal, a very large tract would have a higher probability of being selected than a very small counterpart because the number of disaggregate opportunities was greater in the former than the latter.

A similar problem but different modelling approach to the Lerman study can be found in the research into the joint choice of a residential location and vehicle availability (Sermons and Seredich, 2001). In this study, a large number of alternative residential locations were converted into a manageable choice set using a cluster analysis. The choice structure of households in this model was assumed to be a simultaneous process, namely a joint logit type based on a multinomial logit framework, which the authors admitted that the model could be unrealistic. The adopted choice structure, however, resulted from unsuccessful attempts at calibrating a sequential decision-making procedure. A unique characteristic of this model was that the vehicle availability and a residential location were endogenously treated.

Anas (1981, 1982) applied the random utility model to the research into a combined travel mode and locational choice. In this study, a location choice was supposed as a joint function of locational attractiveness and travel modes available in the location. Four transport modes, which were car, rail, bus and rapid transit, were considered; three categories of data, which were the socio-economic characteristics of a household, the physical and economic attributes of a location and a dwelling, and variables for the features of a combined mode and location, were used. The data used were aggregate. This meant that the model failed to make use of the advantages of the disaggregate model. Nevertheless, Anas argued that the aggregate model was more adaptable to practical prediction, equilibration, and policy analysis because the model had no necessary aggregation step.

As shown in Figure 2-4, the Anas model formulated the three dimensional choice structure of households on a location z , a commuting mode m , and a dwelling type

d . Although the choice dimension in the Anas model was sequential, a nested logit type, the utility function was expressed as a modified joint logit framework.

$$U(z, m, d) = V_z + V_{zm} + V_{zmd} + \varepsilon_{zmd} \quad (2.48)$$

where V is the system components in the utility function and

z, m, d are the characteristics of a location, a transport mode, and a dwelling type respectively.

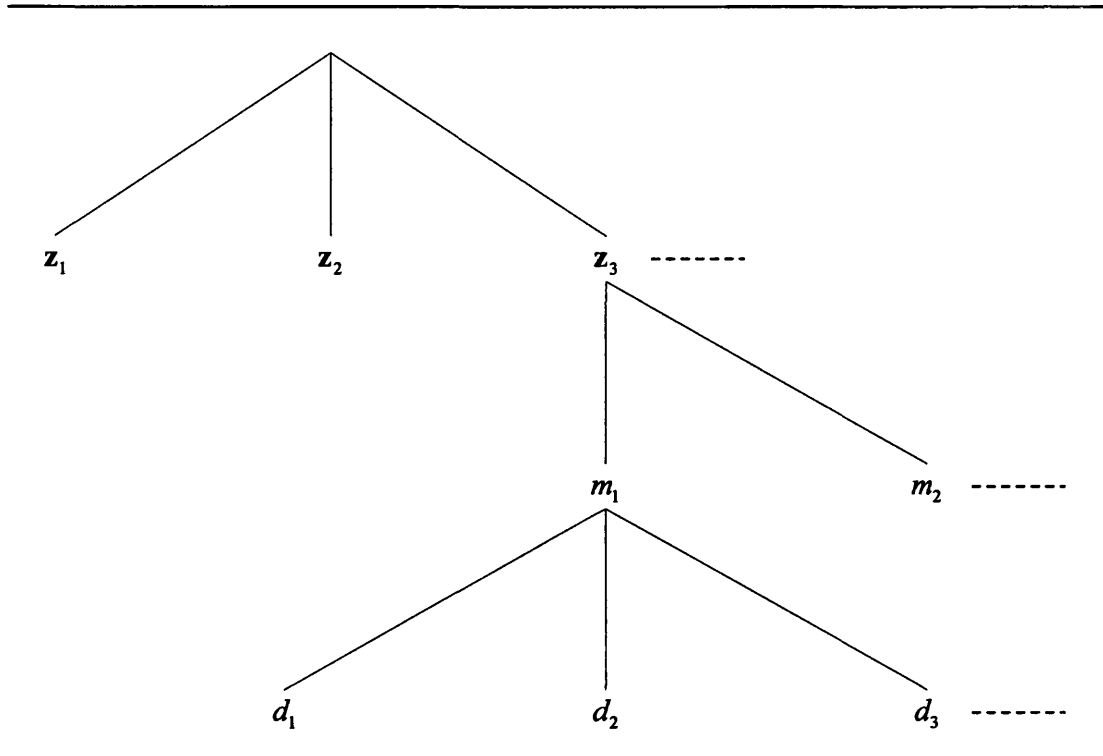


Figure 2-4 The choice structure of the Anas model

The probability of an alternative, which was the bundle of z , m and d , being chosen was calculated using a series of conditional probabilities.

$$\begin{aligned} P(z, m, d) &= P(z) \cdot P(m | z) \cdot P(d | mz) \\ \text{s.t. } P(d | mz) &= 1/D \end{aligned} \quad (2.49)$$

where D is the number of dwellings in a location.

The value of $P(d | mz)$ was assumed to be fixed ratio $1/D$, which suggested that every dwelling was equally chosen. The assumption resulted from the use of the aggregate data. The aggregate data meant that the specific dwelling information was not available in the model. Thus, the Anas model was, in fact, a two-dimensional

choice problem. The other probabilities, $P(\mathbf{z})$ and $P(m|\mathbf{z})$, were determined following the traditional technique of maximum likelihood estimation.

The Anas model was extended into CATLAS (the *Chicago Area Transportation and Land-use Analysis System*; 1981-1985) for the cost-benefit analysis of urban transport investments in Chicago. In the early nineties, CATLAS was extended into NYSIM (the *New York area Simulation Model*; 1990-1993), including non-work travel choices and commercial real estate markets. NYSIM evaluated the impact of urban transport service improvements. Another application of the Anas model was CPHMM (the *Chicago Prototype Housing Market Model*; 1987-1993), which was a dynamic prototype model to investigate the effectiveness, efficiency and inter-temporal effect of the demand and supply sides of housing market assistance policies aimed at improving the welfare of low-income households. After a series of theoretical and empirical researches, which were CATLAS, NYSIM and CPHMM, Anas (1994) developed a commercial package called METROSIM. METROSIM embodied the random utility model with econometrically specified behaviour and a market clearing mechanism. METROSIM has seven sub models: basic industry, non-basic industry, real estate (residential and commercial), vacant land, household, travel demand for commuting and non-work travel, and traffic assignment. These sectors make three equilibriums, which were labour market equilibration and job assignment, housing market equilibrium, and commercial space equilibrium. The equilibriums are achieved by an iterative process adjusting land-use patterns and traffic flows.

The random utility model has been applied to estimate the willingness-to-pay of a household for residential attractiveness. Morisugi and Yoshida (1986) defined the locational attractiveness of residence as the amount of money that a household willingly paid for obtaining a marginal quality of locational attributes. The model claimed that existing logit models failed to measure the willingness-to-pay of a household in two senses; first, the conventional models excluded the budget and time constraints of a household; secondly, the traditional linear-in-parameter function produced an independent value from the income level of a household even though the budget and time constraints were taken into account. An alternative formulation was suggested, which explicitly included the budget and time constraints of a household.

$$\begin{aligned}
& U(\mathbf{x}, \mathbf{z}, Q, t_i) \\
& \text{s.t. } t_i + t_c = t - t_w = t_n \\
& \beta^T \mathbf{z} + H_i = y + H_j
\end{aligned} \tag{2.50}$$

where \mathbf{x} is the vector of composite goods with unit price;
 \mathbf{z} is the vector of housing attributes;
 Q is the level of neighbourhood quality;
 t_i, t_c, t, t_w, t_n are a household's leisure time, commuting time, fixed total time (24hr), working time, and non-working time, respectively;
 H_i, H_j are the housing prices in zones i and j respectively; and
 y is the net income of a household, which is calculated by the annual income minus the revenue from the rent of the owned house.

Another study that stressed the time and budget constraints of a household was found in the *Tool for Integrated analysis of Location and Transport (TILT)* model (Eliasson and Mattsson, 2000). In this study, a household was assumed to choose a bundle of a location, car ownership, and a travel pattern; the travel pattern included trip frequencies, destinations, and travel modes specific to trip types. The overall mathematical structure of TILT was similar to that of a nested logit model; however, an explicit consideration of the time and budget constraints of a decision-maker allowed the model to produce inconstant time and cost sensitivities.

Hunt and others (1993) investigated the influence of critical factors determining the attractiveness of a residential location. The study comprehensively reviewed previous research and reported that the most cited factors for representing this issue were the price of residence and accessibility to a workplace. The stated preference data of households were surveyed. These data were estimated using a multinomial logit model in order to measure the willingness-to-pay of a household. One of the interesting findings was that the value of 'within walking distance of an LRT' was more than eight times the value of 'travel time to work per hour'.

The random utility model has investigated the combination of the locational choice of residence and workplace. Traditionally, workplace locations have been exogenously determined in residential location models. Some empirical studies, however, reported

that the assumption could no longer be supported in a metropolitan area (Hamilton, 1982; Quigley, 1990). In the long run, residential locations would be a conditional function of current workplace locations and vice versa (Abraham and Hunt, 1997).

Anderstig and Mattsson (1991) developed IMREL (*Integrated Model of Residential and Employment Location*), which linked a normative residential location sub model (RES) and a predictive employment sub model (EMP), subject to the upper and lower bounds on the number of households and workplaces. RES was formulated as a sequential choice of residence and a mode to work given a workplace location. RES suggested two types of model, namely a post mode choice form $P(\mathbf{z}) \cdot P(m|\mathbf{z})$ and a pre mode choice form $P(m) \cdot P(\mathbf{z}|m)$. The employment location sub model was a function of accessibility to labour force. EMP reflected the strategic position of an employer because an employer was assumed to compensate the commuting cost of the employee (Jonsson, 2001). Both models were combined in an iterative way. Later, IMREL was reformulated incorporating a network representation (Boyce and Mattsson, 1999). The reformulation produced endogenous travel costs in IMREL.

Waddell (1993) discussed the different decision-making patterns of households; some households may simultaneously decide their residence and workplace; others may sequentially consider the decision. In the sequential cases, some households may choose a residence first and then systematically search for a workplace; others may consider the reverse; since these possibilities could be site-specific, it is important to test all possible cases. Waddell suggested three types of model. The first formulation, a joint logit model, assumed a household simultaneously made a decision. In this case, an alternative was a bundle of workplace, homeownership, and residence. The other two formulations, nested logit models, assumed a household sequentially made a decision. Each one has a two or three-level nested structure. The two-level nested model was expressed as the product of the marginal probability of a workplace location choice $P(w)$ and the conditional probability of the joint choice of homeownership (tenure) and residence $P[(o, z)|w]$. The three-level nested model assumed that a household first chose a workplace location $P(w)$; subsequently a tenure conditional on the selected workplace $P(o|w)$; and finally a residence

conditional on the chosen workplace and homeownership $P(\mathbf{z} | w_o)$. Waddell found that the choice of residence and workplace was jointly determined. In particular, Waddell argued that the spatial dispersion of both residential and workplace locations was so related that a cautious approach was necessary in formulating models.

Abraham and Hunt (1997) suggested an alternative formulation for a combined choice model of residential and workplace locations considering multiple-worker households. The research claimed that a household faces a different dimensional choice problem. In other words, a household chooses a residential location for the whole household and individual household members select their own workplace and travel modes to work. A rational household was supposed to make a compensatory decision, which was the best for the entire household even if the decision was not the optimum for each household member. An alternative utility function was proposed, which represented the trade-off among household members.

$$U = \mathbf{z} + \sum_{h \in H} (w_h + \sigma_h^T m_h) \quad (2.51)$$

where h is an individual household member; the member belongs to a household H , namely $\forall h \in H$;

w_h is the system components of workplaces for a household member h ;

m_h is the system components of travel modes to workplaces for a household member h ; and

σ_h is the inclusive value parameter of age and gender.

In the Abraham and Hunt model, an individual household was assumed to have its own nested choice structure regarding age and gender. A household was expected to choose a combined sub alternative (\mathbf{z}, w) first, and then select a mode m where the inclusive value parameter σ allowed the mode choice of some household members to influence the overall household utility more than that of others.

Most applications of the random utility model in representing the relationship between transport and the location of activities have used observed data, namely revealed preference data (RP). However, RP has been known to produce biases in terms of selecting appropriate variables and generating a choice set. Earnhart (2002)

suggested an alternative model, which combined RP and SP (Stated Preference) data. The research tested three kinds of model: RP only, SP only, and combined RP and SP. The study reported that the result of the combined model was more realistic than that of non-combined models in terms of the significance of parameters. The SP model was also used in estimating values of changes in the negative transport externality such as traffic noise and air quality (Wardman and Bristow, 2004).

2.4.3. Conclusion

Random utility models represent the relationship between transport and the location of activities in terms of the utility maximising behaviour of an individual decision-maker. Early models focused on the theoretical foundation of the framework using a joint or a nested logit approach; later, the models were applied to a wide range of topics: exploring a residential choice process with aggregate data; estimating willingness-to-pay; an explicit consideration of the time and budget constraints of a household; investigating the combined choice of residential and workplace locations; and using combined stated preference and revealed preference data.

This type of model can be a useful tool to represent the nature of the interaction between transport and the location of activities. First of all, the model effectively addresses the locational characteristics. A bundle of locational attributes describes the utility of a location. Each element in the bundle reflects a distinct feature of a location. The random component represents the unobserved characteristics of a location. Secondly, this group of models offers a high degree of behavioural validity of a household's decision-making process. In this type, a reasonable household is assumed to choose a location that offers the maximum utility. This shows that the random utility model naturally describes the relationship between the locational features and the decision-making process of a household. Furthermore, the disaggregate nature of the model addresses the difference in the taste variation of decision-makers. Moreover, the model allows flexible market segmentation by classifying households according to their socio-economic characteristics, which can be linked with multidimensional decision problems. However, the random utility model fails to reflect the interaction between transport and the location of activities. Both transport costs and locational attractiveness are determined outside the model. There is no explicit interaction process between the two factors.

In addition to the discussion regarding the three criteria, the model suffers from difficulty in choice set generation. Any method of grouping alternatives is by its very nature somewhat arbitrary. This means that some levels of abstraction in defining alternatives are required by assumption or the use of reasonable criteria. Furthermore, the aggregation bias is well known to be its critical weakness.

2.5. Bid-rent models

2.5.1. Introduction

Bid-rent models represent the relationship between transport and the location of activities in terms of a bid-auction process of decision-makers for residing at locations. The term *bid-rent*, which was originated from the Alonso (1965) model, is a hybrid concept of the bid and the rent with which Alonso explained an urban land-use process. The *bid* describes the behaviour of decision-makers. Various households are assumed to bid for locations. Landlords are supposed to sell or rent the location to the highest bidder. As a result of this process, the pattern of land-uses and locational values are mutually determined. The *rent* is regarded as the amount of money that the occupant pays or would pay to the use of a unit of a location. The term rent is generic, covering all kinds of market expressions such as contract rents, sales rents, and the costs of ownership. The bid-rent is hypothetical. There is no necessary relationship between the bid-rent and the actual rent that is charged for the use of a location. Therefore, it should be understood that if the rent of a location were paid, an individual would be satisfied to a given degree.

The structure of this type of model adopts a conditional modelling approach:

$$D(\mathbf{h}|\mathbf{z}) \quad (2.52)$$

where $D(\cdot)$ is the demand function for a location market. The formulation is interpreted as that for each location with the vector of characteristics \mathbf{z} , a household type \mathbf{h} who pays the highest rent is expected to occupy the location.

Bid-rent models are reviewed in the next section. The models are broadly divided into deterministic and stochastic types. While the deterministic bid-rent models produce all-or-nothing land-use patterns, the stochastic counterparts create probabilistic variations in land-uses. In the following section, the advantages and disadvantages of this type of model are discussed. The discussion concludes that this group would not meet the three criteria which have been identified in the introductory chapter, even though the model suggests some useful insights into the investigation of the interaction between transportation and the location of activities.

2.5.2. Deterministic bid-rent models

The deterministic bid-rent model, which originated with Alonso (1965), understood a location was as a featureless plain. This meant that all locations were assumed to be of equal quality, namely homogenous. Transport was accessible in all directions. All employment and goods and services were available at the city centre only. Municipal services and tax rates were uniform throughout the city. Locations were freely bought and sold. Both buyers and sellers had perfect knowledge of the market, which meant the price was given and was not affected by the decisions of the buyers and sellers.

The Alonso model dealt with the relationship between locational values and land-uses. The proposed methodology highlighted the role of households on the operation of a location market. In this model, a location market is controlled by a bid-auction process; potential users bid for locations; as a result, locations are assigned to the best bidder. This means that the patterns of land-uses and locational values are mutually determining. This reasoning is distinct, compared with existing approaches. While conventional models suppose that locational values are exogenous and households are price takers, the bid-rent model posits that locational values are endogenously determined and households play an active role to settle locational values.

A modified version of a classical consumer equilibrium theory was proposed using the bid-rent indifference curve of a household. While a classical indifference curve had a combination of goods without considering locational prices, the bid-rent curve consisted of accessibility and locational prices. The model proposed the famous 'double decision process': 'how large a lot he should purchase and how close to the centre of the city he should settle'. This approach was a simplified process with two critical factors, though there are many factors that affect urban land-use patterns.

The budget constraint of an individual was assumed to be the additive form:

$$y = c_l + c_t + c_e \quad (2.53)$$

where y is the income of an individual household and c_l , c_t , c_e are location costs, commuting costs, and all other expenditure respectively; all other expenditure consisted of the total spending on other goods and services including savings

$$c_e = p_1g_1 + p_2g_2 + \cdots + p_ng_n \quad (2.54)$$

where g_i is the quantity of the i^{th} good and p_i is the price of the i^{th} good.

Goods and services were grouped into a composite commodity g for the sake of simplicity; the price of the composite good was given by p_g . Thus, the term of all other expenditure was compactly expressed as $p_g g$.

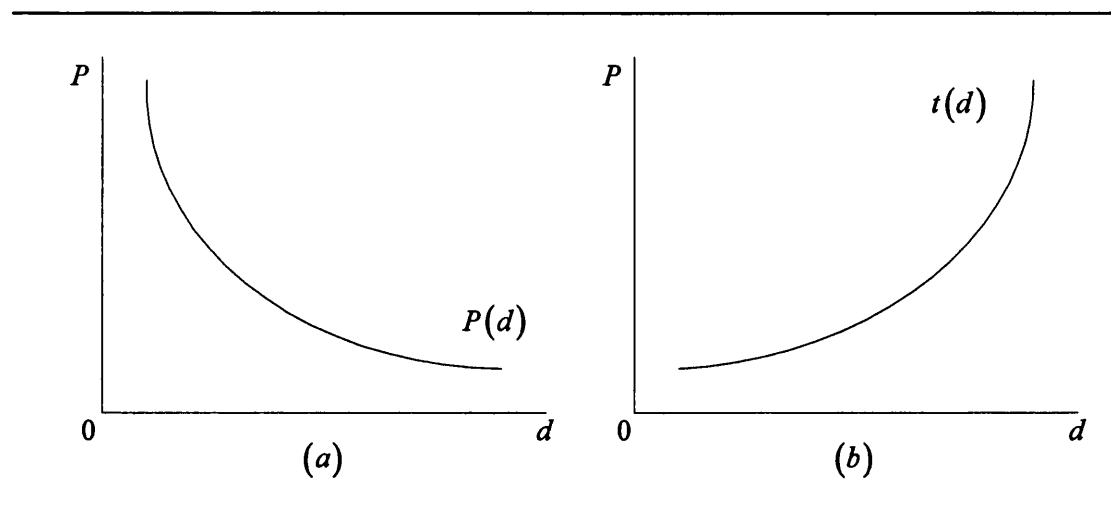


Figure 2-5 A location cost (a) and a commuting cost (b)

Figure 2-5 shows a conceptual diagram for the function of locational and commuting costs in the Alonso model. Location costs were assumed to monotonically decrease with the distance from the centre of the city. An individual chose a location with the quantity of land q ; thus, the land cost was given by $P(d) \cdot q$. Commuting costs $t(d)$ were assumed to monotonically increase with the distance from the centre of the city.

Finally, the budget constraint of an individual was expressed as follows:

$$y = p_g g + P(d) \cdot q + t(d) \quad (2.55)$$

where p_g , g are the price and the quantity of the composite good respectively;

d is the distance from the centre of the city;

$P(d)$ is the price of land at distance d from the centre of the city;

q is the quantity of land; and

$t(d)$ is the commuting cost to distance d .

The utility function of an individual consisted of the quantity of land, the composite good, and distance, which were mapped through indifference surfaces.

$$U = U(\mathbf{g}, q, d) \quad (2.56)$$

An indifference curve represented a constant level of satisfaction. In other words, given any combination of a location and distance, a small increase in distance produced dissatisfaction, and had to be compensated for by a small increase in the quantity of a location; in this process, the level of satisfaction remained the same.

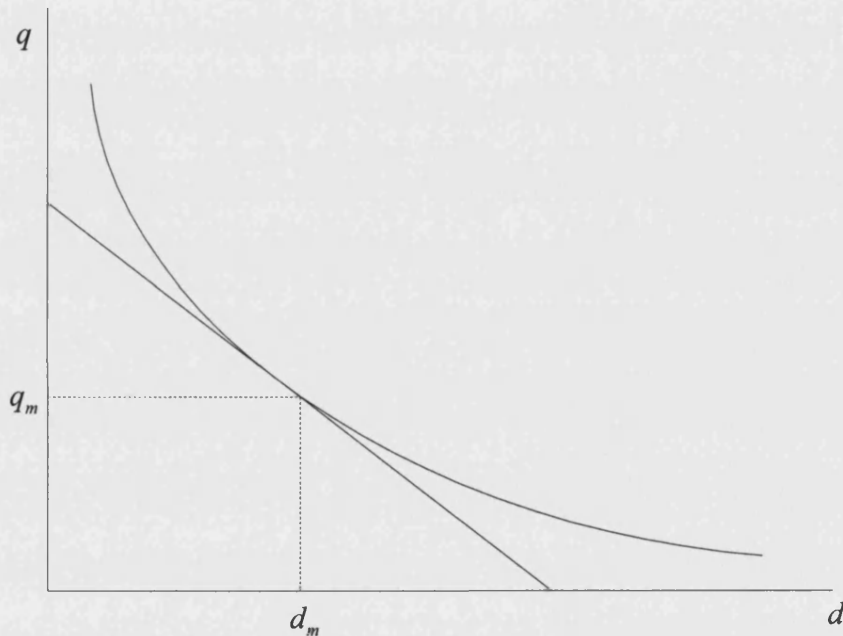


Figure 2-6 A graphical illustration for the solution to the Alonso model

An individual was assumed to maximise utility within the income constraint. This process was represented in terms of the discovery of the combination $(\bar{\mathbf{g}}, q, d)$ that yielded the highest value for U satisfying the budget balance; notice that the composite good was held constant $\bar{\mathbf{g}}$.

Graphically, the process can be illustrated straightforwardly. Figure 2-6 shows the illustration. Let an individual locate at d_m for whatever reason. Equilibrium occurs at the point of tangency between the loci of opportunities and the highest of the

indifference curves. The combination of q and d at this point is the most satisfying to the individual in the choice set that the decision-maker has. Since the indifference surface yields the same satisfaction to an individual, all the combinations of q and d will yield the same value of U . This means that an individual pays at various distances while deriving a constant level of satisfaction.

The Alonso model assumed that the unit of a location was a homogeneous commodity that had a constant value across the market. Hence, in the long run, all locations having the same attractiveness would produce equal service. This meant that locations with the same attraction could be perfect substitutes regardless of the compensating difference between locational characteristics and environmental quality. Obviously, it is unlikely that a household is indifferent from a high quality but small-size location and a low quality but large-size location.

Rosen (1974) incorporated a hedonic theory into the bid-rent model emphasising the inherent heterogeneity of locations. The model stressed that a location is not a homogeneous good but a bundle of attributes $\mathbf{z} = (z_1, \dots, z_I)$ that satisfies the various dimensions of a household's demand (Witte et al., 1979). A hedonic price function $H(\mathbf{z})$ was associated with the bundle. The function guides consumer and producer locational choices regarding the package of characteristics bought and sold. The Rosen model represented the maximum amount that a household would willingly pay for an alternative location having the bundle \mathbf{z} at a given utility, income and taste.

$$\theta = \theta(z_1, \dots, z_I; \bar{U}, y, \alpha) \quad (2.57)$$

where \bar{U} is a given fixed utility level;

$\theta(\cdot)$ the bid-rent function of a household; and

α is the vector of taste variations of a household.

At equilibrium, the bid-rent function of a household is tangent to the hedonic price function $H(\cdot)$ because the function is the minimum price a household must pay in the market. Mathematically, the optimum location in \mathbf{z} -plane occurs where the two surfaces of $H(\cdot)$ and $\theta(\cdot)$ are tangential to each other.

$$\theta(z_1^*, \dots, z_i^*; \bar{U}, y, \alpha) = H(\mathbf{z}^*) \quad (2.58)$$

where $H(\cdot)$ is the hedonic price function of a location market.

The offer function represented the minimum unit price that a firm willingly accepted for the price of housing bundles that the firm produced at constant profits.

$$\phi = \phi(z_1, \dots, z_i; \bar{\pi}, M, \beta) \quad (2.59)$$

where $\phi(\cdot)$ is the offer function of a supplier;

$\bar{\pi}$ is the constant profit level;

M is the level of output of a firm; and

β is the vector of parameters for factor prices and a production function.

At equilibrium, the offer function of a supplier is tangent to the hedonic price function since the function $H(\mathbf{z})$ is the maximum obtainable price in the market. Profits are maximised at tangency between the hedonic price function and the offer function.

$$H(\mathbf{z}^*) = \phi(z_1^*, \dots, z_i^*; \bar{\pi}, M, \beta) \quad (2.60)$$

As discussed above, the market equilibrium occurs at tangency with the hedonic price function. Figure 2-7 shows a one-dimensional graphical expression for the market equilibrium of the Rosen model (1974) where all other attributes except z_i are held constant. Let $Q^d(\mathbf{z})$ be the market quantity demanded for locations with characteristics \mathbf{z} ; $Q^s(\mathbf{z})$ is the market quantity supplied for locations with the same attributes \mathbf{z} ; the market equilibrium occurs at $Q^d(\mathbf{z}) = Q^s(\mathbf{z})$. Even though the market equilibrium can easily be conceptualised, it is not simple to find an actual equilibrium. The difficulty is because the quantities of demand and supply, $Q^d(\mathbf{z})$ and $Q^s(\mathbf{z})$, depend on the hedonic price function $H(\mathbf{z})$. This means that every attribute $z_i, \forall i$ in the hedonic function should also be in equilibrium together with the market equilibrium. Rosen suggested a two-step approximation procedure to overcome the difficulty in finding the double equilibrium. In the first step, the hedonic price function $H(\mathbf{z})$ is estimated; usually, observed prices are regressed against

locational attributes; $\hat{H}(\mathbf{z})$ denotes the resulting function. In the following step, a set of implicit marginal prices is computed regarding the estimated marginal price as the observed marginal price, namely $\partial H / \partial z_i = \partial \hat{H} / \partial z_i$.

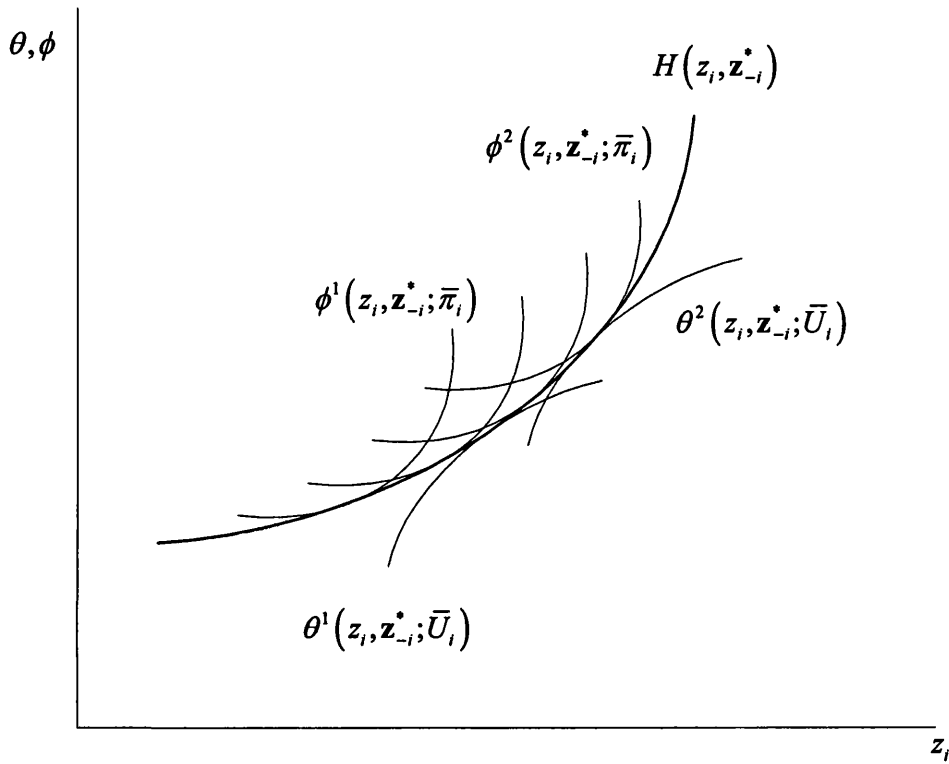


Figure 2-7 The market equilibrium of the Rosen model

The Rosen model was distinct, compared with conventional hedonic price approaches. The model showed the underlying market mechanism while existing approaches only revealed the empirical magnitude of hedonic prices. In the Rosen model, the market price of locations was jointly determined by consumers' evaluating the individual services provided and by the offering price of suppliers for each service. Witte et al. (1979) successfully applied the Rosen result to investigate the rental housing market in non-metropolitan cities in North Carolina, USA. It was demonstrated that the two-step procedure satisfactorily estimated both the bid and the offer functions in the analysis of an implicit housing market. The model confirmed the common hypothesis in which higher status and income households would bid higher for housing quality.

2.5.3. Stochastic bid-rent models

Deterministic bid-rent models addressed the relationship between transport and the location of activities in terms of a bid-auction process. Decision-makers were regarded as bidders and competed to reside. In the end, the best bidder occupied a location. The deterministic model successfully represented the inherent heterogeneity of locations and the decision-making process of individuals. However, the deterministic structure produced an extreme all-or-nothing land-use pattern. This signifies that a certain unit of a location is populated by a single household type. Furthermore, the deterministic nature meant difficulties in representing a number of components of locational attributes and the characteristics of households.

Ellickson (1981) suggested a stochastic bid-rent model that represented the direct specification and estimation of bid-rents for hedonic components (Gross, 1988). The model was a conditional probability approach that could predict that a location having the hedonic bundle \mathbf{z} would be occupied by a household type h , $\forall h \in \mathbf{h}$. The coefficients in the model represented the bid-rent of a household for each locational attribute. Ellickson extended the Rosen formulation adding the component of transport costs in the budget constraint of a household, though the impedance was simply assumed to be the fixed distance to the central business district.

$$\begin{aligned} &U_h(\mathbf{x}, \mathbf{z}) \\ &s.t. \ p_{\mathbf{x}}\mathbf{x} + H(\mathbf{z}) + t(\mathbf{z}) = y \end{aligned} \quad (2.61)$$

where U_h is the utility function of a household type h , $\forall h \in \mathbf{h}$;

\mathbf{z} is a location that is associated with the vector of characteristics; and

$t(\cdot)$ is a transport cost function.

Like the Rosen approach, a bid-rent function of a household was defined, which yielded a certain level of utility \bar{U}_h :

$$\theta = \theta(\mathbf{z}; p_{\mathbf{x}}, y, \bar{U}_h) \quad (2.62)$$

where the price vector $p_{\mathbf{x}}$ was suppressed assuming $p_{\mathbf{x}}$ was invariant throughout the market and the transport cost function $t(\cdot)$ was absorbed in the bid-rent function.

The deterministic bid-rent function was replaced with a stochastic type supposing no household had the same level of perceived income and utility.

$$\theta = \hat{\theta}_h(\mathbf{z}) + \varepsilon_h \quad (2.63)$$

where $\hat{\theta}_h(\cdot)$ is the stochastic bid-rent function of a household type h , $\forall h \in \mathbf{h}$ and ε_h is a random utility term that represents the difference in tastes and income of a household in a group h and unmeasured characteristics of locations.

The probability that a household type h resided at a location \mathbf{z} was given by

$$P(\mathbf{h} | \mathbf{z}) = \text{prob} \left\{ \hat{\theta}_h(\mathbf{z}) + \varepsilon_h > \hat{\theta}_{h'}(\mathbf{z}) + \varepsilon_{h'}, \forall h' \in \mathbf{h}, h \neq h' \right\} \quad (2.64)$$

The formulation in equation (2.64) shows that a location is most likely occupied by the highest bidder. This interpretation is fundamentally the same as the reasoning of the Alonso (1965) approach, namely a location is occupied by the best bidder.

The random term ε_h is supposed as distributed IID Gumbel. Then, the stochastic bid-rent function is represented by a multinomial logit form.

$$P(\mathbf{h} | \mathbf{z}) = \frac{\exp[\hat{\theta}_h(\mathbf{z})]}{\sum_{h'} \exp[\hat{\theta}_{h'}(\mathbf{z})]} \quad (2.65)$$

The functional form of the Ellickson model is seemingly the same as that of a random utility model. The interpretation and the maximisation process, however, are converse between the two models. A random utility model predicts that a consumer chooses a type of a location, namely $P(\mathbf{z} | \mathbf{h})$. In contrast, the stochastic bid-rent model forecasts that a dwelling unit is occupied by a type of consumer, namely $P(\mathbf{h} | \mathbf{z})$. In the Ellickson model, the observation unit is an individual location and the random variable is the bid of each household. Therefore, the maximisation process takes place across alternative bidders, namely households. In contrast, the observation unit in a random utility model is each household that searches locations. In this structure, the random variable is the utility of a household. This means that the maximisation process takes place across the alternative locations (Lerman, 1985; Martínez, 1992b). Although the model interpretation and the maximisation process are the converse of

one another, the two approaches may produce the consistent spatial distribution of households in a competitive market (Martínez, 1992b).

The Ellickson model has been rigorously applied to various research areas. Lee (1982) applied the Ellickson result to the analysis of the intra-urban employment location problem. The study converted a production function into a bid-rent function for capturing the locational behaviour of manufacturing firms. The model predicted the probability of a certain type of firm locating at a site with a specified set of attributes. A similar approach was used in the study of investigating the mismatch between the locational selection of firms and the goal of revitalising areas, Cincinnati USA (Blakley, 1985). Another application of the probabilistic approach was found in the research into representing the relationship between the population movement and the capitalisation of changes in the public sector (Zorn, 1985).

In the Ellickson formulation, the estimated willingness-to-pay of a household was an arbitrary scaling. This was because the model was formulated without considering an observed rent. Hence, the willingness-to-pay of a decision-maker could only be inferred as the ratio of substitution of various attributes. In other words, the slopes of the bid-rent function of attributes were relative to the reference group. Lerman and Kern (1983) added the component of rents actually paid by the winning bidder to estimate the absolute value of the willingness-to-pay.

$$prob[\hat{\theta}_h(\mathbf{z}) + \varepsilon_h = \mathbf{r}^* \text{ and } \hat{\theta}_{h'}(\mathbf{z}) + \varepsilon_{h'} \leq \mathbf{r}^*, \forall h' \in \mathbf{h} \ h' \neq h] \quad (2.66)$$

where \mathbf{r}^* is the vector of the price paid by the best bidder; $\mathbf{r}^* = (\dots, r_j^*, \dots)$.

The term ε_h was assumed to be distributed IID Gumbel:

$$P(\mathbf{h}, \mathbf{r}^* | \mathbf{z}) = f_\varepsilon[\mathbf{r}^* - \hat{\theta}_h(\mathbf{z})] \prod_{h' \neq h} F_\varepsilon[\mathbf{r}^* - \hat{\theta}_{h'}(\mathbf{z})] \quad (2.67)$$

where f_ε and F_ε are the Gumbel cumulative density and distribution functions respectively. In this formulation, the parameters of the bid-rent function were fully identified. This meant that the absolute value of the bid-rent for a locational attribute could be calculated for any type of household.

Gross (1988) empirically tested the Lerman and Kern modification (1983) with the household data from Bogota, Colombia. The model forecast the bid-rent for each attribute of a location and the probability of occupancy of a location by each household type. The results were compared to those of the Follain and Jimenez (1985) that were linked to the hedonic approach. It was concluded that the bid-rent model outperformed the hedonic model in terms of forecasting the demand for a location.

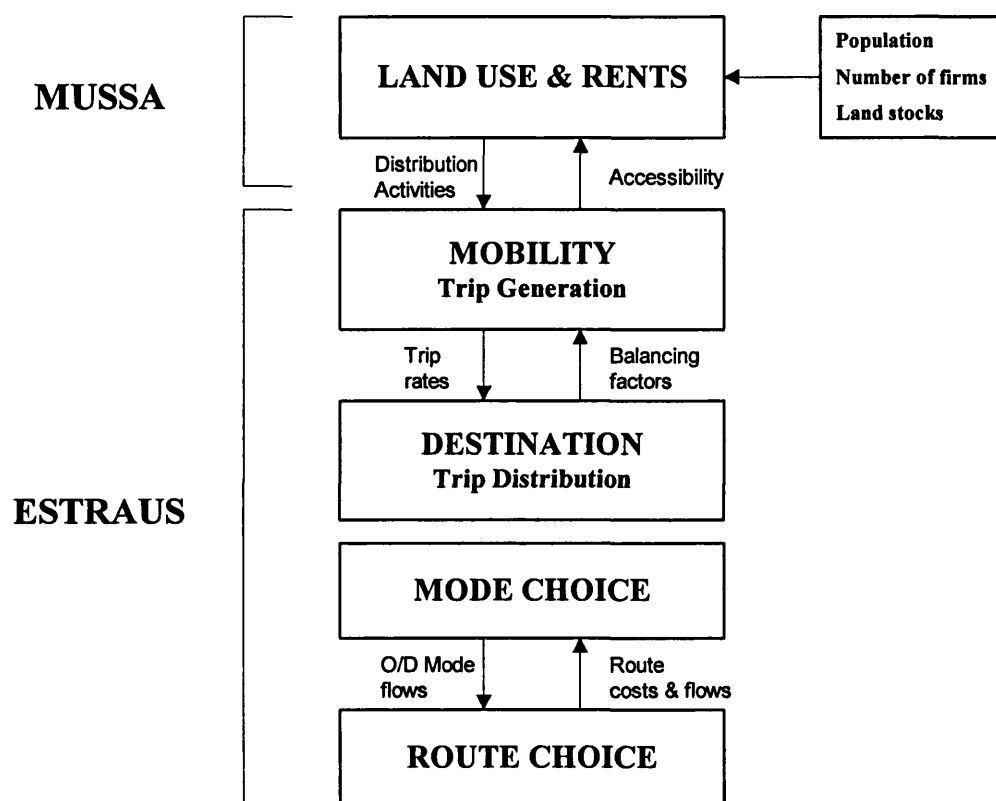


Figure 2-8 The structure of 5-LUT, source: Martínez (1992)

The stochastic bid-rent model has been incorporated into comprehensive land-use transport models. Martínez (1992a, 1996) suggested an alternative transport planning approach, which combined a bid choice land-use model called MUSSA and a transport model called ESTRANUS, namely the 5-stage Land-Use Transport model (5-LUT). 5-LUT assumed that consumers were utility (households) or profit maximisers (firms). This meant that the locational decision of consumers was made by utility or profit maximising constrained to the accessibility level of sites. 5-LUT divided the decision chain of consumers into two components, namely mobility and location. The two factors were necessarily and mutually dependent. The mobility

concerned the number of trips in different purposes. The decision represented the trip generation process that linked trips with socioeconomic variables. The other decision was the location of activities in the space, which was directly related to the land-use model. Martínez argued that traditional transport models did not incorporate a land-use sub-model assuming that trip makers were either job or residential seekers, i.e. either the origin or destination of trips was supposed to be fixed. Even though this assumption offered a convenient perspective in modelling locational advantages and the expected impact of transport facilities, there was no good evidence to judge which end of trips was fixed. Figure 2-8 shows the summary of the process of 5-LUT.

Another interesting application of the stochastic bid-rent model to comprehensive land-use transport studies was the *Random Utility Rent Bidding ANalysis* (RURBAN) (Miyamoto et al., 1992; Miyamoto, 1993; Miyamoto and Udomsri, 1996). RURBAN combined a random utility model $P(\mathbf{z}|\mathbf{h})$ and a bid-rent model $P(\mathbf{h}|\mathbf{z})$ for the quantitative forecasting of land-use changes. RURBAN interpreted a bid-rent model as a locational supply model from the viewpoint of an imaginary landlord; a landlord was defined as a current owner that paid higher rent than any other household. RURBAN was a model for finding general equilibrium between demand and supply of locations. In the supply model, a bid-rent type, households were regarded as random variables that were classified according to socio-economic characteristics. In the demand model, a random utility type, land was segmented into locations that were regarded as discrete options for households. RURBAN sought to find equilibrium between the supply and demand, namely $P(\mathbf{z}|\mathbf{h}) = P(\mathbf{h}|\mathbf{z})$. This framework may offer a unified methodology that could overcome the problem of the partial equilibrium approach in random utility and bid-rent models. Random utility models estimate that each type of household is distributed in proportion to the probabilities with which a location provides the group the highest utility where the value of the location, rent, is given. In contrast, the location share in bid-rent models is proportional to the probabilities that each household type bids the highest rent at location where the level of utility of a household is given (Lerman, 1985; Martínez, 1992b; Miyamoto, 1993). The rents of all locations and the level of utilities of all household types are indispensable. This means that the partial equilibrium approach of the random utility and the bid-rent models may produce undesirable results.

2.5.4. Conclusion

In this section, bid-rent models, which have been applied in representing the relationship between transport and the location of activities, have been reviewed. The bid-rent models address the interaction of decision-makers that seek to occupy locations in terms of a bid-auction process. Starting from Alonso's (1965) the unique trade-off modelling approach, the model has been improved associating a hedonic theory and a stochastic perspective. Recently, the stochastic bid-rent model has been incorporated into comprehensive land-use transport interaction models.

This type of model offers an important insight into investigating the nature of the relationship between transport and the location of activities. First of all, the bid-rent model effectively represents the unique characteristics of locations by incorporating a hedonic theory. In this framework, a location is regarded as globally heterogeneous but composed of aggregate homogeneous components. Each homogeneous element is assumed to be a divisible good that differentiates the bundle of locational characteristics. Thus, hedonic-based bid-rent models satisfactorily overcome the difficulty of addressing the nature of homogeneity and indivisibility of locations. Secondly, the model has a reasonable framework in describing the behaviour of the decision-making process of households. The bidding mechanism of this group has dual advantages: bidding is directly connected to the willingness-to-pay of an individual household; the process rationally represents the interaction between households. Furthermore, the model produces the mutually determined pattern of land-uses and locational values. This is because the best bidder that pays the highest rent is assumed to reside at a location. Hence, the highest rent at locations can be understood as representing locational values and the best bidder is supposed to be an occupant at locations; note that the users and the value of locations are mutually determined. However, it is not easy to establish the explicit interaction between transport and land-use. In most applications of the bid-rent models, transport is merely formulated as one of the components in the budget constraint of a decision-maker. Namely, transport components are represented crudely in this framework.

2.6. Conclusion

This chapter has reviewed models that represent the relationship between transport and the location of activities. Four major fields of models were identified: spatial interaction models, mathematical programming models, random utility models, and bid-rent models. The characteristics of the groups are summarised in Table 2-1.

Table 2-1 Summary of the characteristics of existing models

Model	Locational characteristics	Decision-making process	Interaction
Spatial ¹	Aggregate measure	Crude	Not explicit
Math ²	Aggregate measure	Crude	Partial
Random ³	The utility of attributes	Utility maximisation	Not explicit
Bid-Rent ⁴	The price of attributes	Bidding & utility maximisation	Not explicit
¹ Spatial Interaction Models		³ Random Utility Models	
² Mathematical Programming Models		⁴ Bid-Rent Models	

Each group of models has advantages and disadvantages. The spatial interaction models have the dual advantages of being conceptually simple and a comprehensive nature; however, the models fail to address the unique characteristics of locations and the decision-making process of a household due to the aggregate structure; furthermore, the models crudely represent the interaction between transport and land-use. The mathematical programming models have a simple mathematical form linked to system efficiency; however, the aggregate nature of the model structure means difficulty in representing the systematic properties of locations and the behavioural context of a decision-maker; on the other hand, combined models in this group highlight the interaction between transport and land-use, though the representation is partial. The random utility models consider the unique characteristics of locations using a bundle of components; the models offer a high degree of behavioural validity; however, the process of the interaction between transport and the location of activities is not explicit in this framework. Bid-rent models incorporate a hedonic theory to represent the nature of heterogeneity and indivisibility of locations; the bidding mechanism successfully describes the decision-making process; however, it is not easy to establish the explicit interaction between transport and land-use.

Even though each group of models has a unique mathematical structure, interpretation of the model outcome between the groups would be interrelated. For example, the spatial interaction model and the random utility model would suggest equivalent

results at the aggregate level (Anas, 1983); the Herbert-Stevens model could yield an analogous solution to Alonso's bid-rent model (Herbert and Stevens, 1960) provided that a planner has full information on a location market; the random utility model and the bid-rent model may produce an identical market equilibrium in a perfectly competitive market (Martínez, 1992b). These connections do not mean that the groups of models agree on the responses of the model output. In fact, some studies on comparative responses between different models have reported that the results of models would be diverse even though their collective responses were consistent (Wegener et al., 1991; Mackett, 1991b; Chattopadhyay, 1998). Hence, it would be reasonable to think that the groups of existing models are supplementary rather than opposing each other. However, it should be borne in mind that none of the existing models, either independently or collectively, could satisfactorily meet the three criteria, which have been proposed for the nature of the relationship between transport and the location of activities. A desirable framework should suggest a sound theory representing the locational characteristics, an appropriate perspective on the behaviour of households, and systematic implementation of the interactions between land-use and transportation. As an alternative framework, this research explores a new approach called a bid-rent network equilibrium model.

3. A Bid-rent Network Equilibrium Model

3.1. Introduction

In the introductory chapter, the three criteria that were regarded as the essential nature of the relationship between transport and the location of activities were proposed. The criteria were the locational characteristics, the decision-making process of a household, and the interaction between transport and land-use. In the literature review, existing models were classified into four categories in terms of their mathematical structure. The review examined whether the groups would meet the criteria suggested, but none of the existing groups of models could satisfactorily represent the nature of the interaction between transport and the location of activities.

The purpose of this chapter is to develop an alternative framework that will be called *a bid-rent network equilibrium model* for investigating the relationship between transport and the location of activities. The model is designed to meet the three criteria. In the first place, the unique features of locations are represented using a hedonic price theory. The methodology is known to be a useful framework for investigating commodities that are globally heterogeneous but consist of homogeneous sub-components. Secondly, the model describes the interrelations between decisions within a household and those between households. Modelling a compensatory decision of households will be the primary task for addressing the former, and a non-cooperative competition for the latter. Finally, the interaction between transport and land-use is represented by means of a mutual adjustment process between transport costs and locational benefits. A systematic connection between the three components is embedded in a bi-level mathematical programme.

A conceptual basis for the model is shown in the next section. The basis is considered using a set of criteria that has been suggested for the nature of the relationship between transport and the location of activities. Subsequently, the bid-rent network equilibrium model is derived based on game theory and a bidding mechanism. In the following section, the model extension to a multiclass representation, which is helpful to address inter-class interactions, is considered. Finally, conclusions are presented.

3.2. Conceptual basis of the model

3.2.1. Characteristics of locations

In this study, locations are regarded as commodities that can be consumed. This assumption may suggest that ordinary economic frameworks can be used in the analysis of locations. In traditional market investigations, products are supposed to embody divisible characteristics that together create a homogeneous utility of the goods. This means that two similar products represent no difference in utility rather than means that the two commodities show exactly the same physical features. Consumers are assumed to purchase the group of characteristics in goods even though suppliers and consumers would not effectively recognise the attributes of the products in trade. The bundle of components consumed is used as input that is transformed into utility. Thus, the level of utility of goods depends on the quantity of the characteristics embodied. This process shows that the conventional economic methodologies require two basic assumptions with regard to the features of the goods, namely homogeneity and divisibility. The characteristics of locations, however, violate the two requirements. Indeed, a location is a heterogeneous commodity. There is substantial variation in the structural features, the lot sizes, the transport connections, the quality of local public services, the characteristics of the neighbourhood, and others (Ellickson, 1981). In addition to its heterogeneous nature, a location is an indivisible product. There are observations on spending on locations, but imprecise references to which components are purchased. Furthermore, there is no direct information on the prices of the components embodied in commodities. Thus, the process of production, exchange and consumption of each attribute is implicit. These unique characteristics require an alternative approach. This is mainly because locations are not characterised or approximated by a single price, but represented by a range of prices. The prices depend on the quality of the characteristics that locations contain; note that the prices of goods in the usual economic models depend on the quantity of sub-components.

One of the benchmarks that govern the distinct features of locations is a hedonic price theory. The methodology provides a framework for identifying the structure of prices for the attributes that are embodied in products; then, analyses for demands for goods proceed using the prices of sub-components. In this price model, commodities are

assumed to be globally heterogeneous but are supposed to be composed of aggregate homogeneous attributes. The commodities may not have common prices, but the components are presumed to have at least common price structures. Hence, this approach can be a useful tool to overcome the difficulty in representing the unique nature of locations, namely heterogeneity and indivisibility.

The hedonic price model has been rigorously applied to the studies of the relationship between transport and land-use. Most of the applications have investigated the effects of intrinsic factors in land-use and transport components that determine property values. Three representative directions of research can be enumerated for this branch. In the first place, studies have examined the impact of transport components on the changes in property values. In this group, transport represents either hardware or software in the transport system. In other words, studies have assessed locational premiums associated with accessibility that is considered by either supplies of physical infrastructure or traditional measures for accessibility such as travel time, travel distance, and others (Palmquist, 1982; Al-Mosaind et al., 1993; Lewis-Workman and Brod, 1997; Chen et al., 1998; Jia and Wachs, 1999; Bowes and Ihlanfeldt, 2001; Strand and Vagnes, 2001; Vadali and Sohn, 2001; Weinberger, 2001; Cervero and Duncan, 2002; Srour et al., 2002; Vadali, 2002). The second trend has explored the relationship between the locational value and zonal circumstances. This group includes the interaction between locational prices and either macro factors of the urban spatial system in terms of its size (Jud, 1980; Brasington, 2001) and relative property locations in the system (Archer et al., 1996; Brasington, 2002) or micro locational attributes for the contribution to the locational value (Hughes and Turnbull, 1996; Mills and Simenauer, 1996; Sivitanidou, 1996; Kockelman, 1997; Clapp and Giaccotto, 1998; Downes and Zabel, 2002; Ioannides, 2003). Finally, the third group has dealt with the influences of both transport and locational factors on the property value (Haider and Miller, 2000; Bae et al., 2003).

The effect of externality is another important application of the hedonic price model to transport studies. This branch is related to the strategy for sustainable developments in terms of monetising environmental effects such as noise and emission. In other words, this group seeks to internalise external goods, namely to measure real social

costs (Buseck, 1985; Johnson and Button, 1997; Morrell and Lu, 2000a; Morrell and Lu, 2000b; Lu and Morrell, 2001; Nijland et al., 2003).

The hedonic theory adopted in this study represents locations by means of a vector of objectively measured sub-components

$$\mathbf{z} = (\dots, z_i, \dots), \forall i \in I \quad (3.1)$$

where z_i is the amount of the i^{th} characteristic that differentiates the commodities.

Each z_i is treated as a good. This means that households place either positive or negative marginal values on the component z_i . It is assumed that locations are completely described by the numerical value of the vector of the locational attributes. This value is supposed to offer households distinct packages of locational characteristics and an equivalent utility for locations. There could be many alternative packages because locations have a large number of differentiable attributes.

Each location has a market price so that the location reveals a level of utility

$$\phi(\mathbf{z}) = \phi(\dots, z_i, \dots; \boldsymbol{\alpha}) \quad (3.2)$$

where $\phi(\cdot)$ is a hedonic price function and

$\boldsymbol{\alpha}$ is the vector of parameters for the attributes embodied in a location.

The hedonic price model represents the unique characteristics of locations using a function between locational values and the vector of sub-components. In this framework, consumptions of different locations are represented in which each household purchases a unit of a location with a different amount of the hedonic bundle embodied. Hence, prices for locations ultimately depend on the quantity of the attributes. This means that analyses for locations that are restructured by the hedonic theory can incorporate conventional style economic methodologies: note that the hedonic price model successfully translates the demand determination of locations by the quality of sub-components into by the quantity of sub-attributes. The conversion shows that the hedonic price model is a useful tool to overcome the difficulty in representing the nature of heterogeneity and indivisibility of locations.

For a systematic investigation into the location market, some assumptions are noted:

1. A location market is perfectly competitive. The assumption means that a location is freely bought and sold. This reflects that there are sufficient numbers of suppliers and consumers for locations. Thus, a single household and landowner add no weight to the market. Both the demand and supply sides have perfect information on the market. Reasonable consumers and suppliers try to maximise utility and revenue respectively. They are only constrained by the level of income and the price of the resulting bundle of attributes.
2. There are no better alternative locations outside a study area. No household outside a study area can bid higher than households inside. The bidding mechanism will be discussed in the next section.
3. There is no explicit supply model in this study; instead, the concept of the 'imaginary landlord' (Miyamoto, 1993) is adopted. In this framework, every location is assumed to be owned by its imaginary landlord. When a landlord is an actual user, the landlord is regarded as paying higher rent than any other potential competitor. Imaginary landlords are supposed to supply the location to the maximum bidder. In this case, a hedonic price plays a role as a guide for the locational trade between consumers and suppliers.
4. A location could consist of a number of distinct sub-areas. The characteristics of the sub-areas are not necessarily the same within the whole location. In this case, a location is presumed to reveal a representative attractiveness. Thus, a household is supposed to perceive an average attraction from the location.

3.2.2. Decision-making process of households

An individual household is a decision-making unit for the selection of a residential location. A fundamental perspective for the decision-making issue in this study is that the process is determined in the course of competition for locations. Game theory, combined with a bidding mechanism, is used to represent the process.

Consider a group of households $M = \{\dots, H, \dots\}$ in an area. Some of them are supposed to consider moving their residence. Thus, the decision-makers consist of two distinct subgroups, namely the locating households and the fixed households. The term 'fixed' does not mean that the group does not travel but means that the decision-makers do not change the locations of their activities. Each household is assumed to be rational so that a household is expected to maximise its utility by owning or consuming the hedonic bundle of a location.

Definition 1. A rational household chooses an outcome that maximises its utility when it faces a decision problem.

This study presumes that there are sufficient numbers of households that compete for locations in the system. The sufficiency means that the competition for locations can belong to the group of n-player games if households are regarded as gamers. In the n-person competition, some players may make a coalition to maximise their payoffs. Households, however, are not likely to make a coalition in the process of deciding a residential location. This study assumes that there is no pre-play communication between agents, which means that the game is conducted non-cooperatively. Thus, each household is supposed to maximise its utility without collaborating with others even though the collaboration would produce a higher level of satisfaction.

Definition 2. Non-cooperation is defined as a situation in which no pre-play communication is allowed between households.

Since the game has been assumed to be non-cooperative, interactions between agents play an important role to determine the payoffs of an individual player. This is fundamentally consistent with the assumption of the payoff determination in game theory. In the theory, the action of a player is supposed to depend on actions available

to each agent, the preference of each agent on outcomes, and the speculation of each player about the circumstances of the other gamers with respect to which actions are available to the other players, how the other players rank outcomes, and the belief of the player about the beliefs of the other gamers. Because of this complex situation, a player requires a strategy to win the competition.

Definition 3. A strategy is a complete contingent-plan determining which action a player takes at each information set in which an agent is to move.

A strategy can be either a pure or a mixed type. A player is assumed to have a finite set of alternative strategies. When a player knows exactly what strategies the other gamers take, the agent is expected to choose one definite strategy that can offer the maximum payoff. This case is referred to an agent playing *a pure strategy* game. However, in many cases, a player may not able to exactly guess the action of the other agents. In this case, it is reasonable to suppose that a player takes *a mixed strategy* combining pure strategies with a probability distribution over a set of strategies.

$$S_H = \sum_k \sigma_k s_k = \sigma_1 s_1 + \dots + \sigma_k s_k, \forall H \in M \quad (3.3)$$

where s_k is the k^{th} pure strategy of a player H ;

σ_k is the k^{th} non-negative real number satisfying $\sigma_k \geq 0$, $\sum_k \sigma_k = 1$; and

S_H is a mixed strategy of a player H ; a strategy profile of players is defined as the vector of a mixed strategy of an individual player $S = (\dots, S_H, \dots)$.

Each household normally consists of several members. The residential location may affect their patterns of economic activities. Therefore, the decision to select a residential location would be a compromise that considers the locational requirements of all its members. Nevertheless, most conventional approaches have traced the decision-making process of one household member only. In particular, a head's decision for a workplace location has been dominantly selected in the analyses for this problem. In this framework, the trade-off faced between intra-members could not be treated. Some recent research has suggested an alternative formulation (Waddell, 1993; Jara-Díaz and Martínez, 1999). In these models, a household was presumed to choose its residential location considering a representative individual's primary

location of activity. Subsequently, the other members in the household would choose their locations of activities constrained by the predetermined residential location. This heuristic approach, however, could not satisfactorily represent the systematic strategy-making process of a household. This study assumes that a strategy for competing residential locations is made in a family council considering the trade-off between intra-members' locations of economic activities. Therefore, a reasonable household is supposed to seek the best decision for the entirety, namely a compensatory decision, even if it is not necessarily the optimum for each household member.

This study has assumed that households that join the competition for locations are constrained by their level of income and the hedonic price for the bundle of attributes in locations. It has been also supposed that suppliers of locations offer their properties to a household that willingly pays a higher rent than any other household. These assumptions mean that households that participate in the game for locations face a fair competition. Hence, no household has a more dominant situation than other households. The non-dominance suggests that a household is presumed to make the best response considering the possible strategies of other households.

Definition 4. For any player H in the system, a mixed strategy \mathbb{S}_H^ is the best response to the mixed strategies of the other households \mathbb{S}_{-H} if and only if $U_H(\mathbb{S}_H^*, \mathbb{S}_{-H}) \geq U_H(\mathbb{S}_H, \mathbb{S}_{-H}), \forall H \in M$ where $\mathbb{S}_{-H} = (\dots, \mathbb{S}_{H-1}, \mathbb{S}_{H+1}, \dots)$ and $U_H(\cdot)$ is the payoff function of households in the competition for locations.*

Once a strategy has been made within a household, the household competes for locations against the other households. The competition is complex because there are many potential competitors as well as alternative locations. In order to represent this complexity, the bidding mechanism (Alonso, 1965) is adopted. In this approach, an individual household is regarded as a utility-maximiser. Thus, a household is supposed to search the possible payoffs of alternative locations, and then consume a location that offers the maximum utility. Since potential competitors exist, the location chosen is not always occupied by the household. The uncertainty leads to the process of bidding. A competitor bids for the location consecutively changing their strategy and the amount of the bid. The modifications are affected by those of the other competitors. This causes a candidate to make the best response to the strategies

of the other households. In the end, the highest bidder occupies a given location. This situation is similar to n-person non-cooperative games under the best response strategy; note that the mutual reaction of players is basically non-cooperative and decision-makers are supposed to make an optimal response relative to each other. In this n-player non-cooperative game, the Nash equilibrium can be defined:

Definition 5. A strategy profile $\mathbb{S}^ = (\dots, \mathbb{S}_H^*, \dots)$ is the Nash equilibrium in the n-player non-cooperative game for locations $\theta(\cdot)$ if and only if the best strategy of a household \mathbb{S}_H^* , $\forall H \in M$ is the optimal response to the best strategies of the others \mathbb{S}_{-H}^* , $[\theta(\mathbb{S}_H^*, \mathbb{S}_{-H}^*) \geq \theta(\mathbb{S}_H, \mathbb{S}_{-H}^*)]$, $\forall H \in M$.*

No household has an incentive to deviate from the Nash equilibrium because its bidding strategy represents its optimal response to its perception about the best strategies of the other households. Nash (1951) proved the existence of equilibrium states for any finite non-cooperative game under the mixed-strategy.

Theorem 1. Every finite non-cooperative game has an equilibrium point.

Proof See Nash (1951).

The Nash equilibrium allows for a competitive equilibrium in the system. It has been shown that all households take the best strategy in the competition, which implies that optimal locations are assigned to all households. This is in line with the assumptions in this study regarding the location market and the behaviour of households. In every perfectly competitive market, social welfare is known to be maximised, which is accomplished by the utility-maximising behaviour of the economic agents in supply and demand. Thus, the Nash equilibrium defined is compatible with the assumptions for the design of the bid-rent network equilibrium model.

3.2.3. Interaction between transport and land-use

In general, it is widely accepted that there is a two-way interaction between transport and land-use. Land-use influences travel demands and patterns. The impact of transport on land-use is represented by changes in the level of the accessibility of locations, which in turn affects changes in the location of activities. The location of activities has a cyclical relationship with transport because activities generate travel demands and change travel patterns. Transport and land-use keep exchanging mutual responses with each other. Thus, analyses on transport problems without a proper understanding on the reciprocal relationship cannot be desirable. Nevertheless, conventional approaches have not satisfactorily represented the mutual relationship. In travel demand analyses, locational factors have been considered as parameters that have no explicit feedback with travellers' transport choices. Studies in urban economics have formulated transport impedance as a mere component in budget constraints of decision-makers. These crude representations are apart from a realistic specification of the two-way interaction between transport and land-use.

The interaction is more complex at a micro level. Consider a time period for the process of the interaction between transport and land-use. A household is expected to make diverse choices during the period in terms of transport and the location of activities, which is known as a choice bundle. The choice bundle includes various components such as residence, workplace, the locations of shopping and leisure, car ownership, transport modes, and others. All the decisions in the choice bundle are entangled. A decision chain denotes this relationship. The decision chain is so chaotic that it is difficult to investigate the magnitude of interactions between components and their directions. Hence, every short-term decision of households should be examined when the problem of combined transport and the location of activities is considered. This task is normally referred to as an analysis of urban dynamics. The analysis of urban dynamics requires extensive research into how and when households make these decisions and how current conditions and future opportunities influence the decision-makings. This would be possible theoretically (Abraham and Hunt, 1997).

There have been several attempts to investigate urban dynamics. The first branch is static modelling approaches. This methodology does not explicitly represent urban

dynamics. The static model simulates a single point in time. Hence, the group attempts to predict the urban structure for certain variables, taking other variables as given. Most early-stage models for the relationship between transport and land-use belong to this category. It is obvious that static models are restrictive to representing the urban system in a realistic way. In contrast, quasi-dynamic approaches would be attractive for addressing urban dynamics. The methodology runs for a series of time periods in which changes in transport and land-use are represented by means of successive short-term forecasts. In other words, quasi-dynamic models simplify an entire time period into discrete sub-intervals and simulate some of the relationships within the models responding to variables from the previous time period. Hence, the models use the results from the latest forecast as a baseline for each subsequent projection. These discrete multiperiod dynamics attempt to address the evolution of the urban system. Finally, open-ended approaches are found in some discussions (Hunt and Simmonds, 1993; Wegener, 1994). The framework emphasises constant readjustment in both land-use and transport. The open-ended approach admits that the adjustment could converge towards equilibrium, but argues that the process is limited. This is mainly because physical infrastructure in terms of building stocks and transport system do not instantaneously change. This leads to delays and lags for the adjustment between transport and land-use. Thus, the open-ended methodology regards the urban structure as one which continuously changes but which would never reach equilibrium. This framework is conceptual rather than practical. Thus, it is difficult to find practical examples that are associated with the open-ended approach. To sum up, the static modelling approach would be simple, but can be regarded as one extreme in the sense that the framework does not explicitly represent urban dynamics. The open-ended approach may be another extreme since the methodology emphasises too much detail for the real world. In contrast, quasi-dynamics would be acceptable for representing urban dynamics as an approximation even though the approach does not completely describe the real dynamics between transport and land-use. In fact, real dynamics could be represented in a conceptual framework. In this context, this study adopts a similar approach to quasi-dynamics for modelling urban dynamics.

The approach of quasi-dynamics implicitly assumes that the urban system tends towards a stable equilibrium at a macro level. The methodology also supposes that the macro level equilibrium should accompany a micro level equilibrium. This requires

for this study that households make choices in a reasonable manner. In other words, a household decides one element at a time in the choice bundle, taking into account the current and potential future circumstances of the others (Watterson, 1994). Thus, the time period in this study is regarded as sufficient to allow all households to complete their decisions. In this process, the interaction between transport and land-use is represented in terms of the mutual adjustment between transport impedance and locational attractiveness. This process is considered in the course of competition for locations. As a result of the n-player non-cooperative bidding game for locations, the residential structure with respect to transport costs and locational benefits is changed. A reasonable household is expected to modify its strategy in such a way as to make the best response to other households. The adjusted strategies of households in turn affect changes in transport impedance and locational attractiveness. The adjustment process continues until no household has an incentive to modify its strategy. This state can be referred to as a micro level equilibrium. Even though the micro level equilibrium can be defined in a conceptual framework for the model, it is difficult to find the micro equilibrium in practice because there are many potential players in the system. Furthermore, not all households consider moving residence. The problem is even more difficult when the model treats the order of household locating.

As an alternative, an equivalent bi-level formulation is proposed for an approximate macroscopic solution to this problem. A bi-level mathematical programme is a special case of multilevel optimisation programmes. The multilevel optimisation problems can be defined as mathematical programmes that have a subset of variables constrained to be the solution of a given optimisation problem parameterised by remaining variables. When these problems have two levels, they are referred to as a bi-level mathematical programme. The programme, which seemed to first appear in Bracken and McGill (1973)'s study, has been applied to many areas. This popular application is mainly because the flexible structure of bi-level programmes can systematically represent multilevel decision-makings. In particular, the behavioural interpretation that is motivated by game theory has attracted attention. Four major fields of application can be enumerated in the literature, namely transportation studies, management, general planning, and engineering design (for a comprehensive review, see Vicente and Calamai, 1994; Yang and Bell, 1998; Yang and Bell, 2001).

It should be noted that the following discussion does not explicitly deal with quasi-dynamics. This is because implementing the dynamics requires an investigation into changes in the urban system in both exogenous and endogenous factors; the two factors were described at Chapter Two, Literature Review. In particular, the process of the relocation of economic activities should be considered. In fact, the task is beyond the scope of this study. Urban dynamics could be represented in a framework of comprehensive land-use transport models, which is left for future study.

The proposed formulation consists of finding a solution to the upper level

$$\begin{aligned} \text{Max}(x, y) \quad & F(x, y) \\ \text{s.t.} \quad & g(x, y) \geq 0 \end{aligned} \quad (3.4)$$

where for each value of x , y is the solution to the lower level

$$\begin{aligned} \text{Min}(y) \quad & Y(x, y) \\ \text{s.t.} \quad & h(x, y) \leq 0 \end{aligned} \quad (3.5)$$

where F, f are the objective functions of the upper and the lower levels;

x, y are decision variables of the upper and lower levels respectively; and

g, h are the constraints of the upper and the lower levels respectively.

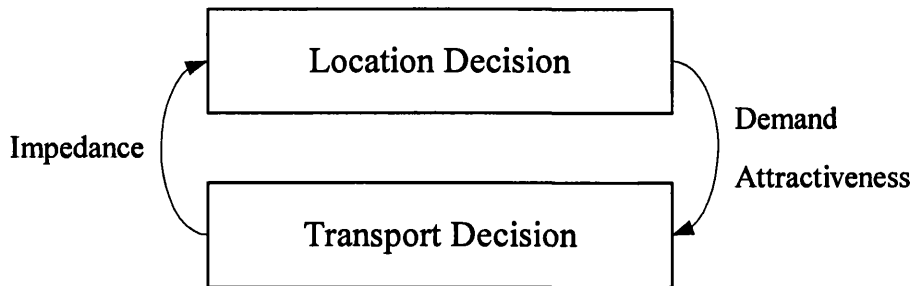


Figure 3-1 A conceptual diagram for the operation of the bid-rent network equilibrium model

Figure 3-1 shows a conceptual diagram for the operation of the proposed formulation. The formulation represents a game involving the newly locating households and those with fixed locations, as discussed in the previous section. The first group is assumed to maximise utility in selecting locations for residence. This process is formulated in the upper level. The behaviour is represented by the decision-makers' consumption of

the hedonic bundles at various locations. The process produces dynamic changes in locational attractiveness and demands. At the lower level, all households choose their routes by minimising transport costs. The level considers all households who are either locators or non-locators. This is because the locators also contribute the determination of travel time in the network. For the inclusion of both groups, the locators are not assumed to move their residence in the lower level. This is because the current residence is supposed to offer the maximum utility to the locating households. Hence, both the locating and non-locating decision-makers are locationally fixed at the lower level. The goal of the lower level decision-makers is the minimisation of transport impedance in spatial interactions. A minimisation of net interaction impedance which is defined as a difference between travel time and locational attraction is the objective of the behaviour of the second group. The minimisation framework addresses a congestible network component as a realistic specification, which determines transport impedance between locations. It is emphasised that the functional relationship between the decision variables of the upper and lower levels establishes endogenously-determined transport impedance and locational attraction, which is one of the unique features of the bid-rent network equilibrium model.

It is useful to note that the overall purpose of this model is to represent changes in urban structure invoked by household locating. The locating behaviour is specified with respect to the utility maximisation of households in selecting residential locations. The optimisation problem requires an equilibrium transport solution from the lower level. Of course, the lower level also requires the solution of the upper level in the form of locational attractiveness. However, the transport solution is not the ultimate target that the proposed model wants to find. Thus, it can be regarded that the overall structure of the model has a form of bi-level optimisation programmes rather than combined equilibrium models that can be found in some existing approaches.

This formulation translates the n -person non-cooperative competition for locations into a duopolistic Cournot game; the game of the many-to-many competition is replaced by that of the one-to-one case, namely the game between the groups of locators and non-locators. It was demonstrated that one-to-one games formulated in a bi-level mathematical programme offer a macroscopic solution to n -player games

when the number of gamers involved is large (Bell and Cassir, 2002). This means that the formulation suggests a macroscopic solution to the locational competition since the number of households has been assumed to be sufficient. In this duopolistic Cournot game, the Nash equilibrium is redefined as follows:

Definition 6. A strategy profile $\mathbb{S}^ = (\bar{\mathbb{S}}^*, \tilde{\mathbb{S}}^*)$ is the Nash equilibrium in the duopolistic Cournot game for locations $\theta(\cdot)$ if and only if $\theta(\bar{\mathbb{S}}^*, \tilde{\mathbb{S}}^*) \geq \theta(\bar{\mathbb{S}}, \tilde{\mathbb{S}}^*)$ and $\theta(\bar{\mathbb{S}}^*, \tilde{\mathbb{S}}^*) \geq \theta(\bar{\mathbb{S}}^*, \tilde{\mathbb{S}})$ where $\bar{\mathbb{S}}^*$ and $\tilde{\mathbb{S}}^*$ are the strategy profiles of the locating and the fixed households respectively.*

The Nash equilibrium of the duopolistic Cournot competition successfully reformulates that of the n-player non-cooperative game. The reformulation, however, implicitly assumes that a single representative player for each group can explain the diverse behaviour of households within the specific groups. The restriction can be mitigated by disaggregating households into several classes that are assumed to show homogeneous decision-making process. The modification converts the duopolistic Cournot game into an oligopolistic Cournot competition for locations.

Definition 7. A strategy profile of household classes in the system $\mathbb{S}^ = (\dots, \mathbb{S}_m^*, \dots)$, $\forall m \in M$ is the Nash equilibrium of the oligopolistic Cournot game for locations $\theta(\cdot)$ if and only if \mathbb{S}_m^* is the best response of a household class m to the optimal strategies of the other classes $\mathbb{S}_{-m}^* = (\dots, \mathbb{S}_{m-1}^*, \mathbb{S}_{m+1}^*, \dots)$, namely $\theta(\mathbb{S}_m^*, \mathbb{S}_{-m}^*) \geq \theta(\mathbb{S}_m, \mathbb{S}_{-m}^*)$, $\forall m \in M$.*

Some applications of the Cournot game to transport studies can be found in the literature (Kita, 1999; Bell, 2000; Yang et al., 2001; Bell and Cassir, 2002). Many studies, however, have adopted the framework of the Stackelberg game; for comprehensive reviews of the application, see Yang and Bell (1998, 2001). It is useful to distinguish the Cournot game from the Stackelberg game for a clearer specification of the model formulation. There are similarities and dissimilarities between the two games. Both games deal with small numbers of gamers and investigate quantity competitions. In these games, no player has a dominant strategy; hence, the best-response analysis can be applied to investigate the behaviour of decision-makers. The main difference between the two games is observed in the order of gamers' actions.

While the Cournot game assumes that all players act simultaneously, the Stackelberg game supposes a sequential decision-making process. In the Stackelberg game, one player acts before the others. The first mover is referred to as a leader and the others are denoted as followers; for this reason, the Stackelberg game is referred to as a leader-follower game. Therefore, the leader has more information than the followers whilst players in the Cournot game have the same degree of information. For further discussion about the two games, see Appendix I-Fundamentals of Game Theory.

The difference between the Cournot game and the Stackelberg game is clearer from a mathematical viewpoint. In general bi-level mathematical programmes, for each value of the upper level variable x , the constraints of the lower level define the constraint set of the lower level problem $\Omega(x)$:

$$\Omega(x) = \{y : h(x, y) \leq 0\} \quad (3.6)$$

The set of solutions for the lower level problem $W(x)$ is given by minimising the lower level function $f(\cdot)$ for all the values in $\Omega(x)$ of the lower level variable y :

$$W(x) = \left[y : y \in \arg \min \{f(x, y) : y \in \Omega(x)\} \right] \quad (3.7)$$

Given these definitions, the bi-level problem can be reformulated as follows:

$$\begin{aligned} & \text{Max}(x, y) \quad F(x, y) \\ & \text{s.t.} \quad g(x, y) \leq 0 \\ & \quad \quad y \in W(x) \end{aligned} \quad (3.8)$$

where the feasible set $\{(x, y) : g(x, y) \geq 0, y \in W(x)\}$ of the bi-level mathematical programme is called the induced or inducible region. The induced region is usually nonconvex, which means that the bi-level programme may have multiple solutions.

The Cournot game seeks to find a mutually consistent solution. Most solution-find mechanisms in this group solve the problem by successively alternating levels, exchanging solutions with each other. Thus, the formulation can be regarded as effectively representing the mutual adjustment process in terms of an iterative modification of transport impedance and locational attractiveness.

$$\begin{aligned}
& \text{Max}(x, y) \quad F(x, y) \\
& \text{s.t.} \quad y \in W(x) \\
& \quad \quad x \in W(y)
\end{aligned} \tag{3.9}$$

where $W(y) = [x : x \in \arg \min \{F(x, y) : x \in \Omega(y)\}]$ and $\Omega(y) = \{x : g(x, y) \geq 0\}$.

In contrast, the formulation of the Stackelberg game explicitly represents the leader-follower structure. The upper level normally suggests the leader's problem, and the lower level addresses that of the follower.

$$\begin{aligned}
& \text{Max}(x, y) \quad F(x, W(x)) \\
& \text{s.t.} \quad y \in W(x)
\end{aligned} \tag{3.10}$$

The Stackelberg formulation shows two distinct features. First, the model solves the follower's problem with the solution to the leader's problem given, which describes the first-mover advantage. Secondly, the formulation calculates the leader's problem without fixing the follower's decision variable, which represents the follower's reaction. These two mechanisms are the unique characteristics of the Stackelberg game; for further discussion about the characteristics, see Appendix I-Fundamentals of Game Theory. The Stackelberg formulation has been popularly used for investigating network design problems; see Yang and Bell (1998, 2001). For further mathematical comparison between the two formulations, see Heydecker (1986).

3.2.4. Discussion

This section has described the conceptual basis for the development of the bid-rent network equilibrium model. The discussion proceeded in relation to the three criteria for the nature of the interaction between transport and the location of activities. First, the unique characteristics of locations were represented by means of incorporating a hedonic price theory. The approach was expected to overcome the difficulty in representing the heterogeneity and the indivisibility of locations. Secondly, a game theoretical framework, associated with a bidding mechanism, was adopted for modelling the decision-making process of households. The methodology proposed was thought to satisfactorily represent the decision-making process not only within a household but also between households. Finally, a mutual adjustment process was used for addressing the interaction between transport and land-use. The process produces endogenously-determined transport impedance and locational attractiveness. The three components were embodied in a bi-level mathematical programme. The programme in the final formulation suggested an approximate oligopolistic solution to the n -player non-cooperative game. The systematic connection between the theories on the relationship between transport and the location of activities and the methodologies for the practical modelling would offer little doubt the model, which will be developed in the next section, could be an alternative framework for the analysis of the interaction between transport and the location of activities.

It is worth noting the disagreement between the design of the model and location markets in the real world. This study has assumed that the location market is perfectly competitive; there are no regulations in trades of locations; there exist sufficient numbers of suppliers and consumers; the two agents in demand and supply have perfect information on the market. In general, the level of social welfare is maximised in a perfectly competitive market. This means that the model represents a competitive equilibrium, which allows for optimal locations for all households. The allocation is in line with the Nash equilibrium defined. The real world, however, does not always reflect the situation designed in the model. There exists a lot of friction for households in moving residence. This is mainly because locations are usually traded in private markets that involve diverse uncertain factors. Information on the market is anything but perfect and is costly to obtain. Government may impose regulations on the market.

Thus, the residential structure in the real world is not necessarily optimal for households. Even though this study will incorporate a stochastic component for mitigating the disagreement, the difference between the model outcome and the real urban structure is unlikely to be completely resolved. Nevertheless, it should be borne in mind that a modelling is a means of describing general trends. This means that a modelling would not represent all related aspects. The process inevitably requires some assumptions. The assumption of this study in terms of a competitive location market would be acceptable for representing the relationship between transport and the location of activities. This is because a system exhibits its general trend in a macroscopic scale when the number of agents involved is large (Fujita, 1989, p. 2). This is one of the reasons why this study adopts the bi-level formulation that suggests an oligopolistic approximation. Another important point is that over realistic considerations in modelling do not necessarily create better outcomes. Specifications of much detail might have an advantageous position in describing structures for the real world. This process, however, inevitably involves errors in terms of the measurement of related factors and their functional relationship. Furthermore, the specification requires significant resources in terms of money and time.

3.3. The model

3.3.1. Introduction

This section presents the process for developing a new model of the relationship between transport and the location of activities, namely a bid-rent network equilibrium model. The model systematically represents the three criteria of the nature for this relationship. The criteria involve the unique characteristics of locations, the composite decision-making process of households, and the interaction between transport and land-use. The fundamental assumption for the modelling is that this issue can be understood in the course of competition for locations. As discussed in the previous section, this competition is equivalent to an n -player non-cooperative game. Since the Nash equilibrium can be defined in this type of competition, the resulting model is expected to suggest equilibrium locations that are optimal for households. The optimal locations eventually represent an efficient urban structure in a study area.

A general framework for the model of the relationship between transport and the location of activities is explored in the next section. The framework is considered in terms of competition for residential locations, which results in an equivalent n -player non-cooperative game. In the following two sections, two components in the general framework are specified, namely transport and location decisions. The transport decision is a model for investigating the behaviour of non-locators. The behaviour is represented using a framework of a minimisation of net interaction impedance. A hedonic-based random bid-rent model is suggested for the description of the behaviour of locators. The locating behaviour is discussed by means of representing the locational decision of households. Subsequently, a bi-level mathematical programme that combines the two components is considered. The formulation is regarded as an oligopolistic approximation to the n -person non-cooperative game. The resulting model produces endogenously determined transport impedance and locational attractiveness, which are regarded as a unique feature of the bid-rent network equilibrium model. Finally, some concluding remarks in terms of a model extension are presented. The extension deals with an incorporation of a multiclass framework, which establishes a multiclass bid-rent network equilibrium model.

3.3.2. General framework

In this section, a general framework for modelling the relationship between transport and the location of activities is considered. As discussed in the section on the conceptual basis for the model, the process of the interaction is investigated in terms of competition for locations. An n-player non-cooperative game is the resulting formulation for the general framework of this relationship.

Consider a study area. The study area is presumed to have sufficiently large numbers of households and residential locations. The sufficiency suggests that a continuous modelling approach can be feasible for representing the relationship. The continuous framework can produce an approximating solution to this problem even though the nature of households and locations are obviously discrete. Some of the households are assumed to consider moving their residence during a time period that is appropriate for this process; the time period is specific to the study area. Hence, the households are classified into two distinct decision-making groups, namely the locating households and the fixed households. The households in each group are sub-divided into distinctive classes m , $\forall m \subset M$ whose behaviour is supposed to be homogenous in the context of the interaction between transport and the location of activities.

As discussed in the conceptual basis of the model, an individual household in the locating group maximises payoffs in the selection of a residential location. This process is conducted non-cooperatively, which causes competition for locations. The payoffs of a household in the competition can be represented by means of a joint function of private goods, locational attractiveness, and transport costs.

$$U_{H \in m}^{r\sigma}(\mathbf{g}, \varphi^r, u_H^{r\sigma}; \beta) \quad (3.11)$$

where $U(\cdot)$ is the utility function of a household in the competition for locations;

\mathbf{g} is the vector of private goods, namely $\mathbf{g} = (\dots, g_j, \dots)$, $\forall j \in J$;

φ^r is the value of a location r ; the value is functionalised with the hedonic vector \mathbf{z} ; $\mathbf{z} = (\dots, z_i, \dots)$, $\forall i \in I$;

$u_H^{r\sigma}$ is the minimum transport cost of a household H that belongs to a household class m , $\forall m \subset M$; the cost is calculated summing the

minimum transport cost of each members between a residential location r and their activity locations s , namely $u_H^{r\sigma} = \sum_{h \in H} u_h^{rs}$; and

β is the vector of parameters that is associated with the private goods g , the locational value φ^r , and the transport impedance $u_H^{r\sigma}$.

Some additional clarification is given for the notation. First, the transport impedance in the utility function is represented with respect to monetary costs in order to be compatible with the locational attractiveness and the private goods. For the consistency, a parameter that converts the transport costs into monetary costs is needed. In this study, the transport costs are specified without the parameter assuming the costs are specified in the equivalent monetary costs. This setting is for notational simplicity. Secondly, the transport costs are represented as the impedance between a household's single residential location r and its members' locations of primary activities s ; σ denotes the set of destinations chosen by the household members $\{s(h)\}$, $\forall h \in H$. This is related to the modelling assumption about the compensatory decision; a household is assumed to seek the best decision for its entirety even though the decision is not necessarily the optimum for the individual household members; for further discussion about the compensatory decision, see the section on the conceptual basis of the model. The primary activity is defined as regular spatial interactions that can include journeys to workplace, shopping, school, and other purposes. The location of primary activity is fixed in this study. Thus, the notation rs could also be denoted as $r|s$, which means a residential location r conditional on the primary activity location s . Similar rules are applied to the following equations. One more point should be mentioned is that a household is supposed to require a positive amount of the private goods, the locational value, and the transport costs. This means that the three components are essential in describing the behaviour of households in this competition. Thus, the utility function is defined with positive values of the three attributes. The positive values, however, do not mean that all the components produce a positive marginal utility. In fact, the greater the transport cost, the less the level of utility. This is a reason why the parameter vector is specified for the components. Thus, the parameter for the transport cost is expected to be negative values.

The budget constraint of an individual household is expressed as an additive function:

$$\sum_j \rho_j g_j + \phi(\mathbf{z} | r) + \sum_{h \in H} u_h^{rs} = y_H \quad (3.12)$$

where ρ_j is the price of a private commodity g_j ;

$\phi(\mathbf{z} | r)$ is a hedonic price function for a residential location r ; and

y_H is the income of a household H .

The budget constraint shows that all income is used for the consumption of the three components, which implies that this formulation does not consider the savings of households. Another interesting point in the budget constraint is that the transport cost is considered for the total cost of all household members rather than just the cost for one member. Again, this is related to the hypothesis of the compensatory decision.

With the utility function and the budget constraint, a general formulation is given for the relationship between transport and residential location. The formulation is represented in terms of the utility maximisation of a household with respect to the private goods, the locational value, and the transport impedance.

$$\begin{aligned} & \text{Max } U_{H \in m}^{r\sigma}(\mathbf{g}, \varphi^r, u_H^{r\sigma}; \beta) \\ & \text{s.t. } \sum_j \rho_j g_j + \phi(\mathbf{z} | r) + \sum_{h \in H} u_h^{rs} = y_H \\ & \quad g_j \geq 0, \varphi^r \geq 0, u_h^{rs} \geq 0, z_i \geq 0, \forall h, i, j, r, s \end{aligned} \quad (3.13)$$

It is assumed that the market for the private goods is perfectly competitive, which means that the private goods and their price are invariant throughout the market. Thus, the component can be suppressed and the model can be reformulated more compactly with respect to the locational attractiveness and the transport cost:

$$\begin{aligned} & \text{Max } U_{H \in m}^{r\sigma}(\varphi^r, u_H^{r\sigma}; \beta) \\ & \text{s.t. } \phi(\mathbf{z} | r) + \sum_{h \in H} u_h^{rs} = y_H \\ & \quad \varphi^r \geq 0, u_h^{rs} \geq 0, z_i \geq 0, \forall h, i, r, s \end{aligned} \quad (3.14)$$

The general formulation for the n-player non-cooperative competition consists of two decision variables, which are the transport cost and the locational attraction. The formulation shows that the issue is a composite decision-making process involving the

two components. The structure of the general formulation seems to allow for the application of classical optimisation approaches to find an equilibrium. The approaches, however, would not be feasible straightforwardly in an attempt to find a micro level equilibrium. The difficulty is mainly because there are many potential competitors and alternative locations in the competition. Furthermore, two distinct behaviours of households are captured in this problem. As discussed in the section on the conceptual basis of the model, households are either locators or non-locators. The behaviour of each group is clearly differentiated in the context of this relationship. The locators face the decision regarding moving their residence. On the other hand, the non-locators are only interested in minimising transport impedance. Thus, the formulation is divided into two sub-components, namely a transport decision which represents the decision-making process of non-locators, and a location decision which refers to the behaviour of locators. It should be noted that the transport decision considers all households that contribute the determination of transport costs in the system. Thus, both the locating and non-locating households are included, but the transport decision is denoted as the problem of non-locators. This is because households are not assumed to change residence in representing the transport decision, even though the problem includes the locating group. The two components are combined using a bi-level mathematical programme. This formulation is equivalent to an oligopolistic Cournot game for approximating the n-person non-cooperative competition for locations. The resulting formulation is regarded as overcoming the difficulty in investigating the distinct behaviour of locators and non-locators, and representing a micro level equilibrium in an acceptable manner.

3.3.3. Transport decision: the problem of non-locators

The transport decision addresses the behaviour of non-locators in the competition for residential locations. The discussion traces the composite decision-making process of households for maximising locational benefits and minimising transport costs. The resulting formulation is an equivalent minimisation of the net interaction impedance. The minimisation establishes the lower level of the bid-rent network equilibrium model. This section begins with a preliminary description on a network representation. Subsequently, the transport decision is explored.

Consider a mathematical network $G = (N, A)$ where N represents a set of nodes and A denotes a set of directed links. Let $n \in N$ be an individual node and $a \in A$ be an individual link in the network. Let $R, R \subseteq N$ denote a set of origin centroids at which flows are generated and $S, S \subseteq N$ a set of destination centroids at which flows are terminated. The origin and destination nodes are not mutually exclusive, namely $R \cap S \neq \emptyset$. This means that some nodes can serve as origins and destinations.

In the network, there are nonnegative demands for travel $Q = [q^{rs} : \forall r \in R, \forall s \in S]$ where q^{rs} denotes the flow of spatial interactions between an origin-destination pair r and s . The OD flow includes both the locating travellers \bar{q}^{rs} and the locationally fixed decision-makers \tilde{q}^{rs} , thus $q^{rs} = \bar{q}^{rs} + \tilde{q}^{rs}$. It is also noted that the spatial interactions are represented by the unit of household members, which allows the members travel to their primary activity locations; note that the destination is represented as s rather than σ ; the primary activities have been defined as regular spatial interactions in this study. Again, the multiple destinations of a household are related to the modelling assumption about the compensatory decision. One spatial interaction flow is represented as one vehicle in the network during a peak period, presuming one vehicle is occupied by one household member only. Because of this assumption of vehicle occupancy, transport costs could be unsatisfactorily calculated. The impedance, however, is expected to reveal a general transport cost structure in a large area under a long-term perspective with which this model deals.

Table 3-1 Summary of the notation

N	a set of nodes
A	a set of links (arcs)
R	a set of origin nodes; $R \subseteq N$
S	a set of destination nodes; $S \subseteq N$
q^{rs}	the flow of spatial interactions between an origin-destination pair r and s , $\forall r \in R, \forall s \in S$;
P^{rs}	a set of paths connecting r and s , $\forall r \in R, \forall s \in S$;
f_p^{rs}	the flow on a path p connecting an origin-destination pair r and s , $\forall p \in P^{rs}$;
c_p^{rs}	the transport cost on a path p between an origin-destination pair r and s ;
v_a	the flow on a link a , $\forall a \in A$; $\mathbf{v} = (\dots, v_a, \dots)$
t_a	the transport cost on a link a ; $\mathbf{t} = (\dots, t_a, \dots)$
δ_{ap}^{rs}	an indicator variable: $\delta_{ap}^{rs} = 1$, if a link a is on a path p , and 0 otherwise

The demands are distributed among allowable paths $p \in P^{rs}$. Let f_p^{rs} be the flow on a path p connecting an OD pair r and s . The conservation equation between the path flow and the flow of spatial interactions is given by

$$q^{rs} = \sum_p f_p^{rs}, \quad \forall r \in R, \forall s \in S \quad (3.15)$$

where a nonnegative path flow that satisfies the conservation equation is feasible and a *feasible flow pattern* is defined as the family of an individual feasible flow, $\Omega = [f_p^{rs} : \forall p \in P^{rs}, \forall r \in R, \forall s \in S, f_p^{rs} \geq 0]$.

Given a path flow f_p^{rs} , a volume on a link a is given by

$$v_a = \sum_p f_p^{rs} \cdot \delta_{ap}^{rs}, \quad \forall a \in A \quad (3.16)$$

where δ_{ap}^{rs} is an indicator factor: $\delta_{ap}^{rs} = 1$, if a link a is on a path p , and 0 otherwise.

Let t_a be the transport cost of each link a . It is assumed that the cost t_a is the function of a link volume v_a . In other words, the link cost is flow-dependent.

$$t_a = t_a(v_a), \quad \forall a \in A \quad (3.17)$$

The transport cost on a path p is the sum of the cost of links that form the path

$$c_p^{rs} = \sum_a t_a \cdot \delta_{ap}^{rs} \quad \forall p \in P^{rs}, \forall r \in R, \forall s \in S \quad (3.18)$$

The functions of transport impedance $t_a(\cdot)$ are set to $C = [t_a(v_a): \forall a \in A]$. Finally, the general transport network combines the directed mathematical network G , the travel demand Q , and the transport cost function C , namely $N = [G, Q, C]$.

It has been assumed that the location of the economic activity of each traveller is known and fixed. It has been also assumed that the decision-makers in the transport decision are locationally fixed. This means that the origin of each trip-maker is fixed. Therefore, the origin-destination matrix is known in the transport decision. Given the OD matrix, the network behaviour of the trip-makers determines the equilibrium path flow $\{f_p^{rs,*}\}$. The path flow is used to calculate transport impedance between an OD pair $\{r, s\}$, which is required at the upper level to represent the locating behaviour of households that is an ultimate target of the bid-rent network equilibrium model.

For representing the decision of non-locators, this study explicitly deals with two network performance indices of locational benefits and transport costs. Households might be assumed to choose residential locations considering the locational attractiveness only. This means that the factor of travel time does not affect the decision of households regarding the location of activities. This is far from a realistic network representation. Another approach might assume that households choose the residence that could minimise transport impedance without explicit consideration of the locational factor. This branch is self-contradictory because the framework addresses the decision on residential locations without representing the residential attractiveness. Thus, it would be reasonable to assume that a household seeks a composite goal that has the two components, namely choosing a location with the highest attractiveness while requiring the least transport cost. The decision, therefore,

can be understood to be the result of a trade-off between locational attractiveness and transport impedance (Alonso, 1965; Sheffi, 1985, p. 165).

Each residential location r is associated with locational attractiveness $\bar{\varphi}^r$, which reflects an activity opportunity available there. It has been presumed that households do not change residence in the transport decision. It has been also supposed that the current residence offers the highest attractiveness for households. The value of the locational attraction is calculated from the maximum bid-rent, which is derived from the willingness-to-pay of households; a derivation detailed for this factor will be discussed in the next section. The maximum bid-rent is interpreted as the unit prices that are normally used in urban economics for addressing the attractiveness of locations; a practical example of this interpretation is found in Chapter Five, Case Studies. In the transport decision, the attractiveness is associated with the value of travel time in order to be compatible with the network travel times.

A net interaction impedance for representing the composite decision is defined as the difference between transport costs and locational attractiveness:

$$|c_p^{rs} - \bar{\varphi}^r| \quad (3.19)$$

where the locational factor $\bar{\varphi}^r$ is specified in the same unit of transport costs, as the transport impedance is specified in the equivalent monetary unit in the utility function. These settings have been suggested for notational convenience.

The definition of the net interaction impedance suggests a clear representation concerning the behaviour of non-locators. In this framework, each household is supposed to choose the location with the lowest net interaction impedance. In other words, locations are chosen so that the difference between transport impedance and locational attractiveness $|c_p^{rs} - \bar{\varphi}^r|$ is minimised. Thus, at equilibrium, all locations chosen have the same net interaction impedance which is equal to or less than that of the other locations which have not been chosen. Even though this definition is conceptually feasible, there could be time inconsistency between the two components. This is because trip frequency is normally observed over a shorter term than a rental period. To tackle this problem, this study assumes that transport costs are an average

value that is perceived during the rental period. Based on these definitions and assumptions, the equilibrium condition can be defined as follows:

$$|c_p^{rs} - \bar{\varphi}^r| \begin{cases} = \eta^{rs} & \text{if } f_p^{rs*} > 0, \forall r, s \\ \geq \eta^{rs} & \text{if } f_p^{rs*} = 0, \forall r, s \end{cases} \quad (3.20)$$

where η^{rs} is the minimal net interaction impedance and

f_p^{rs*} is the equilibrium path flow.

A formulation using variational inequality is suggested to represent the equilibrium condition. In the first place, a path cost function is extended:

$$\tilde{c}_p^{rs} = \tilde{c}_p^{rs}(f_p^{rs*}, \bar{\varphi}^r) = \sum_a t_a \cdot \delta_{ap}^{rs} - \bar{\varphi}^r, \forall p, r, s \quad (3.21)$$

where the extended cost function $\tilde{c}_p^{rs}(\cdot)$ is link-flow dependent since the cost $t_a(\cdot)$ has been assumed to be the function of link flows.

The minimisation of the net interaction impedance can be formulated by finding the vector of an equilibrium path flow, namely $\{f_p^{rs*}\}, \forall f_p^{rs} \in \Omega$:

$$\begin{aligned} \sum_{rs} \sum_p \tilde{c}_p^{rs}(f_p^{rs*}, \bar{\varphi}^r)(f_p^{rs} - f_p^{rs*}) &\geq 0 \\ \text{s.t. } \bar{\varphi}^r &= E[Max U_{H \in m}^{r\sigma}(\cdot)] \quad \forall r \\ \sum_p f_p^{rs} &= q^{rs} \quad \forall r, s \\ f_p^{rs} &\geq 0 \quad \forall p, r, s \end{aligned} \quad (3.22)$$

where Ω is the set of feasible solutions, namely $\Omega = [f_p^{rs} : \forall p \in P^{rs}, \forall r \in R, \forall s \in S]$

and $E[Max U(\cdot)]$ means the maximum expected bid-rent that is determined at the upper level; the detail is discussed in the next section. The equilibrium flow found is used to evaluate travel time between each OD pair. The travel cost is required at the upper level to represent the behaviour of locators.

The variational inequality shows that, for a given equilibrium flow, any deviations from the equilibrium cannot reduce the net interaction impedance. In other words, no trip-maker would be tempted to reduce his impedance changing his path because no alternative route could offer lower costs; this is consistent with the Nash equilibrium.

Another point to be noted in the formulation is that every formulation of a path-based variational inequality can be converted into that of a link-based counterpart (Smith, 1979). A path-based formulation, however, is adopted in this study mainly because a path-based model is more general than a link-based formulation. The two formulations are equivalent if and only if transport costs are assumed to be additive and path costs consist of a set of link travel times only (Bliemer, 2001).

The general condition of the existence and uniqueness of a solution to the problem of variational inequality has been established (Smith, 1979; Dafermos, 1980). Here, more efficient versions of the conditions are provided. The variational inequality has at least one solution provided that the set of feasible path flows is non-empty (see Bell and Iida, 1997, pp. 86-89). The variational inequality has a unique equilibrium solution in terms of link flows when the Jacobian of the link cost functions with respect to link flows are positive definite (see also Bell and Iida, 1997, pp. 86-89).

Even though there is a unique vector of equilibrium link flows, there are many feasible path flow vectors in general. Thus, it is difficult to define the vector of equilibrium path flows uniquely in the formulation; note that the formulation is path-flow based. However, this property implies no difficulty in defining a unique value of the net interaction impedance that is the key modelling hypothesis for representing the transport decisions of non-locators. This is because the unique path costs can be defined, though there are many candidate vectors of equilibrium path flows.

To sum up, this section has discussed the decision of non-locators in the interaction between transport and the location of activities. The minimisation of the net interaction impedance is an equivalent resulting framework. The variational inequality formulation satisfactorily represents the Nash equilibrium, which is regarded as a successful modelling of the transport decision in the competition for locations.

3.3.4. Location decision: the problem of locators

In this section, the behaviour of locators, which is the counterpart to the model for non-locators, is considered. The locating behaviour is represented in terms of the locational decision of households in the competition for residence. A hedonic-based random bid-rent model is suggested for the formulation, which establishes the upper level of the bid-rent network equilibrium model. This framework is regarded as an oligopolistic approximation to the n-player non-cooperative game.

In the section on the general framework, a household is assumed to consist of several members. Each member is allowed to have his or her own primary activity location. This economic activity location of each household member is assumed to be known and fixed. Thus, a destination demand constraint is required.

$$\sum_h \sum_r \sum_m h_m^{rs} = \bar{D}^s \quad (3.23)$$

where \bar{D}^s is the total number of demands attracted to a primary activity location s .

The general formulation of this competition has been suggested in terms of the utility maximisation of locators. The solution to this problem can be represented using an indirect utility function following standard economic methodologies.

$$\begin{aligned} & \text{Max } U_{H \in m}^{r\sigma}(\varphi^r, u_H^{r\sigma}, y_H - \phi(z|r) - u_H^{r\sigma*}; \beta) \\ & \text{s.t. } u_H^{r\sigma*} = \sum_{h \in H} \tilde{c}_p^{rs}(h; f_p^{rs*}, \bar{\varphi}^r) \quad \forall H, r, s \\ & f_p^{rs} \geq 0, \varphi^r \geq 0, u_H^{r\sigma} \geq 0, z_i \geq 0 \quad \forall H, i, p, r, \sigma \end{aligned} \quad (3.24)$$

where $u_H^{r\sigma*}$ is the minimal transport cost of a household. The impedance has been endogenously determined in the transport decision. As noted in the previous section, the transport cost is specified in the equivalent monetary unit.

The indirect utility function would suggest a connection between the behaviour of a household and the locational characteristics. Specifically, the formulation links the utility maximising behaviour of a household with the locational value, which is determined by the hedonic bundle z . The linkage, however, is indirect because the hedonic bundle z affects the decision-making process via the hedonic price function $\phi(\cdot)$. For an explicit representation between the two components, the bid-rent of a

household is defined as a function of the locational characteristics \mathbf{z} ; this yields a certain level of utility U^0 (Alonso, 1965; Rosen, 1974).

$$\theta_H(\mathbf{z} | r; U^0) \quad (3.25)$$

where $\theta_H(\cdot)$ is the bid-rent function of a household with respect to the hedonic attributes of residential characteristics \mathbf{z} .

The bid-rent function substitutes the hedonic price function to address the explicit linkage between the behaviour of a household and the locational properties.

$$\begin{aligned} \text{Max } U_{H \in m}^{r\sigma}(\varphi^r, u_H^{r\sigma}, y_H - \theta_H(\mathbf{z} | r) - u_H^{r\sigma*}; \beta) &= U^0 \\ \text{s.t. } u_H^{r\sigma*} &= \sum_{h \in H} \tilde{c}_p^{rs}(h; f_p^{rs*}, \bar{\varphi}^r) \quad \forall H, r, s \\ f_p^{rs} &\geq 0, \varphi^r \geq 0, u_H^{r\sigma} \geq 0, z_i \geq 0 \quad \forall H, i, p, r, \sigma \end{aligned} \quad (3.26)$$

The bid-rent function is interpreted as the willingness-to-pay function of a household on the locational characteristics. The interpretation is because the function $\theta_H(\cdot)$ is represented in terms of the hedonic vector \mathbf{z} . The level of utility U^0 is monotonic with the net income of a household $(y_H - \theta_H(\cdot) - u_H^{r\sigma*})$. Thus, the bid-rent function can be inverted with the utility function of a household.

$$\begin{aligned} \theta_H(\mathbf{z} | r) &= \text{Max } U_{H \in m}^{r\sigma}(\varphi^r, u_H^{r\sigma}, y_H - u_H^{r\sigma*}, U^0; \beta) \\ \text{s.t. } u_H^{r\sigma*} &= \sum_{h \in H} \tilde{c}_p^{rs}(h; f_p^{rs*}, \bar{\varphi}^r) \quad \forall H, r, s \\ f_p^{rs*} &\geq 0, \varphi^r \geq 0, u_H^{r\sigma} \geq 0, z_i \geq 0 \quad \forall H, i, p, r, \sigma \end{aligned} \quad (3.27)$$

This formulation is similar to that of Alonso-Rosen's deterministic bid-rent function. The formulation represents the inherent heterogeneity as well as the indivisibility of locations. The model also describes the decision-making process of a household. Furthermore, the framework systematically connects the unique characteristics of locations and the behaviour of a household. The deterministic model, however, produces an extreme all-or-nothing land-use pattern; an entire unit of a location is populated by a single household class, which is unrealistic. The deterministic problem can be mitigated incorporating a stochastic framework. The converting process is straightforward. None of the households have the same perceived net income

$(y_H - u_H^{r\sigma*})$. No households value the utility level U^0 to the same degree. Hence, the two factors can be substituted by a stochastic component (Ellickson, 1981).

$$\begin{aligned}\theta_H(\mathbf{z} | r) &= \text{Max}_{H \in m} U_{H \in m}^{r\sigma}(\varphi^r, u_H^{r\sigma}; \beta) \\ &= \text{Max} \left[\beta^T V_{H \in m}^{r\sigma}(\varphi^r, u_H^{r\sigma}) + \varepsilon_{H \in m}^{r\sigma} \right] \\ \text{s.t. } \varphi^r &\geq 0, u_H^{r\sigma} \geq 0, z_i \geq 0 \quad \forall H, i, r, \sigma\end{aligned}\tag{3.28}$$

where $V_{H \in m}^{r\sigma}$ is a system component in the utility function; and

$\varepsilon_{H \in m}^{r\sigma}$ is a probabilistic component that explains an individual household's different perception on the net income and the utility level.

In principle, various functional forms can be considered to represent the hedonic-based stochastic bid-rent model. A particularly interesting model type is a discrete choice framework. This methodology presumes that the random component $\varepsilon_{H \in m}^{r\sigma}$ is distributed as IID Gumbel. Then, the formulation for the locational competition of households can be expressed by a multinomial logit formula.

$$\begin{aligned}Pr^{rs}(\mathbf{m} | \mathbf{z}) &= Pr(U_{H \in m}^{r\sigma} \geq U_{H' \in m'}^{r\sigma}), \forall m \in M \\ &= \frac{\exp[\beta^T V_{H \in m}^{r\sigma}(\varphi^r, u_H^{r\sigma})]}{\sum_{m'} \exp[\beta^T V_{H \in m'}^{r\sigma}(\varphi^r, u_H^{r\sigma})]}\end{aligned}\tag{3.29}$$

where the parameter vector can be estimated using the standard maximum-likelihood technique; the procedure is presented in the next chapter, Solution Algorithm.

The multinomial logit representation suggests the probability that a location with the hedonic bundle \mathbf{z} is occupied by a household class m . This probability is used to calculate the spatial interaction flows of the locating households.

$$\bar{q}^{rs} = Pr^{rs}(\cdot) \times \bar{D}^s\tag{3.30}$$

where the demands together with the flows of non-locating households $\tilde{q}^{rs}, \forall r, s$ constitute the origin-destination matrix $\{q^{rs}\}$ that is required at the lower level.

The supply side of locations has been assumed to offer a location to the household that pays the highest rent. Thus, this formulation produces mutually determined patterns of locational share and locational value. The mutual determination is because

the bidder who pays the highest rent would reside at a location. The formulation can be regarded as an oligopolistic approximation to the n-player competition for residential locations. This is because the multinomial logit framework can be interpreted as a representation of a class-to-class behaviour rather than that of an individual-to-individual. In addition, the behaviour of classes is supposed to take the optimal reaction to each other non-cooperatively. This means that the framework of the best response can be applied to analyse this competition. Thus, the resulting formulation can be regarded as equivalent to an oligopolistic Cournot game; for more discussion about the Cournot game, see Appendix I-Fundamentals of Game Theory.

In this formulation, the expected maximum rent is directly calculated by the highest bid, which is determined by the willingness-to-pay of households. In this study, the maximum bid-rent is understood as unit price that is normally interpreted as a zonal attraction measure in urban economics. Thus, the highest bid is interpreted as locational attractiveness. The value can be calculated by a logsum formula:

$$\bar{\varphi}^r = E \left[\text{Max}_{H \in m} U_{H \in m}^{r\sigma}(\cdot) \right] = \ln \sum_{m'} \exp \left[\beta^T V_{H \in m'}^{r\sigma}(\cdot) \right] \quad (3.31)$$

where none of households perceive a location to have the same value. A household is assumed to place an individual valuation on locations, namely $(\varphi^r = \bar{\varphi}^r / y_H)$, $\forall r$.

It is useful to note that the multinomial logit representation of the hedonic-based random bid-rent model seems to be identical to the structure of a random utility model. However, the interpretation and maximisation processes are converse between the two models. Figure 3-2 shows a conceptual comparison between random utility and stochastic bid-rent models. In a random utility model, a household is assumed to compare locations and choose one specific location that offers the maximum utility. Therefore, the maximisation process takes place across alternative locations. In contrast, a bid-rent model is interpreted as representing a location with characteristics z being occupied by a household class m , $\forall m \subset M$ that bids the highest rent for the location. In other words, a bid-rent model suggests that a location is populated by the bidder who is willing to pay a higher rent than any other household class. Therefore, the maximisation process takes places across household classes.

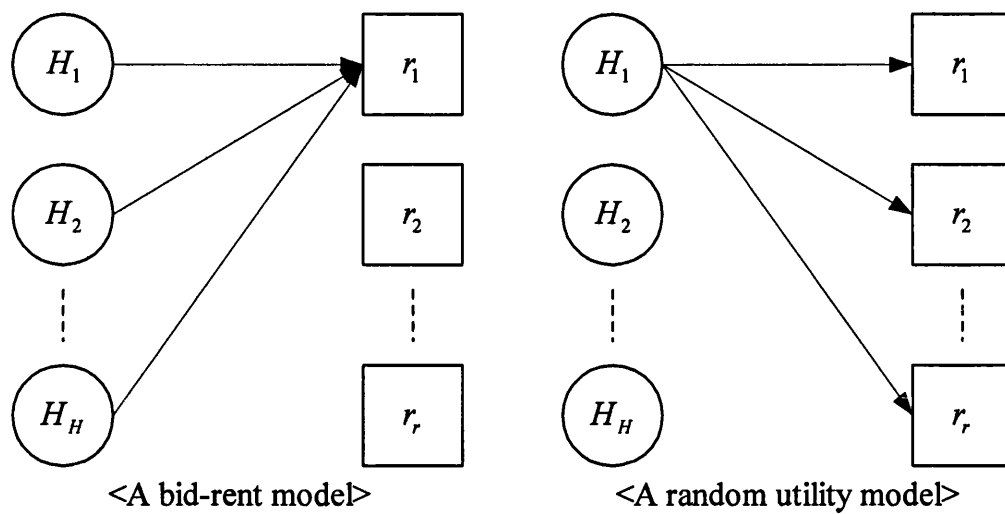


Figure 3-2 A conceptual comparison between bid-rent and random utility models

Even though the difference in terms of the model interpretation and the maximisation process between random utility and traditional probabilistic bid-rent models is obvious, the two models would produce a consistent residential structure in a perfectly competitive market (Martínez, 1992b). This means that a well specified class structure in a random utility model would suggest a similar outcome to a conventional stochastic bid-rent model. The mutual similarity, however, would not happen in the bid-rent network equilibrium model. This is because the model explicitly internalises the congestible network and systematically functionalises the network performance indices of transport impedance and locational attractiveness. Thus, the two network performance indicators, which identify the urban structure, are endogenous in the bid-rent network equilibrium model. In contrast, as stated in Chapter Two, transport costs are exogenous in random utility and bid-rent models, though locational benefits might be endogenous; there is no explicit interaction process between the transport impedance and the locational attractiveness in the two conventional approaches.

In summary, this section has discussed the problem of locators, which has been represented in terms of the locational decision of households in the competition for residential locations. The hedonic-based random bid-rent function has been suggested for describing the nature of the competition. The function represents the relationship between the decision-making process of households and the unique characteristics of locations. The resulting formulation is equivalent to an oligopolistic Cournot game.

3.3.5. Formulation

In the previous sections, the frameworks for representing the behaviour of locators and non-locators were discussed. The model for non-locators was explored in terms of the composite decision-making process for minimising transport impedance and maximising locational attractiveness. The minimisation of the net interaction impedance was suggested for addressing the behaviour of non-locators. The model for locators was investigated using the hedonic-based random bid-rent function. This formulation systematically described the relationship between the characteristics of locations and the decision-making process of households. The resulting formulation was understood as equivalent to an oligopolistic Cournot competition.

One of the fundamental assumptions of this model is that the location of primary activities of each household member is known. This means that the total level of demand attracted to each economic activity location is known and fixed. Hence, in representing locating behaviour, the issue was which residential location a household would choose. In other words, the target was to find the demand at the origin of their work trip. In contrast, since decision-makers in the lower level were locationally fixed, the origin-destination flows $\{q^{rs}\}$, $\forall r \in R, \forall s \in S$ are known. Thus, the issue was to find equilibrium travel time between locations that is required at the upper level.

In this section, a bi-level structure, combining the models for the behaviour of locators and non-locators, is proposed, namely *the bid-rent network equilibrium model*. Let $Pr^{rs}(\cdot)$ be the share of the total number of locating households between a residential location r and a primary activity location s , $\forall r \in R, \forall s \in S$; $0 \leq Pr^{rs}(\cdot) \leq 1$ and $\sum_r Pr^{rs}(\cdot) = 1$. Let \mathbf{z} be the hedonic vector of locational characteristics, thus $\mathbf{z} = (\dots, z_i, \dots)$, $\forall i \in I$ and $U_{H \in m}^{r\sigma}$ be the utility function of a household H in the competition for locations, namely $U_{H \in m}^{r\sigma} = U_{H \in m}^{r\sigma}(\varphi^r, u_H^{r\sigma}; \beta)$. Let \bar{D}^s be the total number of household members attracted at an economic activity location s ; each member h belongs to a locating household H , namely $\forall h \in H$.

$$\begin{aligned}
Pr^{rs}(m|\mathbf{z}) &= Pr(U_{H \in m}^{r\sigma} \geq U_{H' \in m'}^{r\sigma}) & \forall m \subset M \\
s.t. \quad \bar{\varphi}^r &= \bar{\varphi}^r / y_H & \forall r \\
u_H^{r\sigma} &= \sum_{h \in H} \tilde{c}_p^{rs}(h; f_p^{rs*}, \bar{\varphi}^r) & \forall H, r, s \\
\sum_h \sum_r \sum_m h_m^{rs} &= \bar{D}^s & \forall s \\
f_p^{rs} \geq 0, \bar{\varphi}^r \geq 0, u_H^{r\sigma} \geq 0, z_i \geq 0 & \forall H, i, p, r, \sigma
\end{aligned} \tag{3.32}$$

where $u_H^{r\sigma}$ is obtained by finding the equilibrium flow $\{f_p^{rs*}\}$, $\forall f_p^{rs*} \in \Omega$ using an origin-destination matrix that is calculated as follows: $\{q^{rs}\} = \{\tilde{q}^{rs} + Pr^{rs}(\cdot) \times \bar{D}^s\}$

$$\begin{aligned}
\sum_{rs} \sum_p \tilde{c}_p^{rs}(f_p^{rs*}, \bar{\varphi}^r)(f_p^{rs} - f_p^{rs*}) &\geq 0 \\
s.t. \quad \bar{\varphi}^r &= E(\text{Max} U_{H \in m}^{r\sigma}) & \forall r \\
\sum_p f_p^{rs} &= q^{rs} & \forall r, s \\
f_p^{rs} &\geq 0 & \forall p, r, s
\end{aligned} \tag{3.33}$$

where Ω is the set of feasible solutions, namely $\Omega = [f_p^{rs} : \forall p \in P^{rs}, \forall r \in R, \forall s \in S]$ and \tilde{q}^{rs} is the fixed number of trips between an origin-destination pair $\{r, s\}$ generated by the non-locating decision-makers.

The upper level represents the behaviour of the locating households in terms of the bidding process of decision-makers. This is directly connected to the willingness-to-pay of the households for residential locations. The willingness-to-pay is used to determine the locational attractiveness $\bar{\varphi}^r$. This level also determines the probability of the spatial demand of locators between an OD pair $\{r, s\}$, namely $Pr^{rs}(\cdot)$. The probability is used to update the origin-destination matrix $\{q^{rs}\}$ that is required at the lower level. The formulation in the lower level explicitly considers the two network performance indices of locational attractiveness and transport impedance. The lower level evaluates the transport cost between locations, which is used at the upper level.

The functional relationship between the two components in the bi-level structure produces endogenously determined transport costs and locational benefits. The endogenous factors are a unique feature of the bid-rent network equilibrium model. Discussion for the endogenous solutions can be found in the literature. In urban

economics, some studies have specified transport components as an endogenous factor that contributes to the determination of urban land-use (see Fujita (1989) and references therein). This group has regarded traffic congestion as an important negative externality in the system and extended the traditional trade-off modelling approach between transport and housing by explicitly specifying commuting costs as an endogenous factor. The studies in urban economics have claimed that transport costs depend on the amount of land used for the space of transport infrastructure, and argued that congestion charges as a format of location tax would be recommendable for an efficient allocation of land. Even though this approach might offer a useful insight into the treatment of transport costs as a realistic specification, the framework would not be desirable. This is mainly because the methodology suggested does not deal with the behaviour of traffic flow. Specifically, the approach does not consider the complex relationship between traffic flow and transport impedance. On the other hand, the combined model, which is a mathematical programming model, emphasises flow-dependent transport costs. As discussed in the review of existing models, the group has integrated traffic assignment models into spatial interaction frameworks for a realistic network representation. The combined model eventually produces endogenously determined transport costs. This approach, however, is partial because the locational attractiveness, which is the counterpart to transport impedance, is assumed to be exogenous. Random utility models may suggest an endogenously determined locational attractiveness in the format of accessibility (see Ben-Akiva and Lerman, 1987, pp. 300-304). The expected maximum utility, which is an output from the modelling approach of this group, may be interpreted as a measure for locational benefits. Random bid-rent models also suggest a measure of the locational attractiveness. The expected maximum bid-rent in this group offers significant insights into understanding the attractiveness; this study has incorporated the component for determining the locational attractiveness. The two approaches, however, are partial for evaluating the endogenous network performance indices. This is mainly because the transport cost in these groups is assumed to be exogenous. Some comprehensive land-use transport interaction models such as LILT (Mackett, 1983), ITLUP (Putman, 1983, 1991), and METROSIM (Anas, 1994) have suggested an integrated framework to determine the two network performance indicators. The stability, however, is crudely achieved iterating extensive sub-components until the models converge. This means that the approach requires large amount of time in the

adjustment and, more critically, the two indicators could be unstable. Another critical problem is that the models could be inoperational. This is because the sub-models demand huge quantity of data and systematic model operation processes such as parameter estimation, model calibration, and model validation. Thus, the extensive models would not be attractive for representing the endogenous determination.

To sum up, the existing approaches for dealing with the endogenous network performance indices are either partial or crude. In contrast, the bid-rent network equilibrium model is attractive in two senses: first, the two indicators in the proposed model are endogenously determined in a single unified framework, which addresses a stable equilibrium; secondly, the model suggests a systematic functional relationship between transport impedance and locational attractiveness requiring reasonable resources. These characteristics would mean that the bid-rent network equilibrium model could be a desirable modelling approach for producing the endogenous indices.

Another important point that should be noted in the issue of the endogenous solutions is a connection with a multiple period adjustment in the competition. As a result of the non-cooperative bidding game, the residential structure with respect to transport costs and locational benefits would change. A reasonable household would modify its strategy in such a way as to make the best response to the decisions of the other households. The adjusted strategy of households in turn affects transport impedance and locational attractiveness. This process continues until no household has an incentive to modify its strategy. The bi-level formulation represents this process in terms of a mutual adjustment between transport costs and locational attraction. This process, in the end, produces the endogenous network performance indices.

3.3.6. Discussion

This section has discussed the process for the development of the bid-rent network equilibrium model. The process was considered in terms of the n-player non-cooperative game of households. The goal of the framework was to represent the three structural nature of the relationship between transport and the location of activities, and was to find the Nash equilibrium, which was understood as suggesting optimal locations for households. The group of the optimal locations eventually represented an efficient urban structure. Two distinct behaviours, which were the travel behaviour of all households and the locational behaviour of households those changing residence over the time period, were identified in the process of the competition. The behaviour of non-locators was represented using the minimisation of the net interaction impedance. The hedonic-based random bid-rent function was suggested for describing the behaviour of locators. The two components were combined in a bi-level programme. The formulation was interpreted as an oligopolistic Cournot game that was an approximation to the n-player non-cooperative competition. The resulting formulation was expected to produce endogenously determined transport impedance and locational attractiveness. The characteristics were regarded as an important contribution of this model to the studies of land-use transport interactions.

The bid-rent network equilibrium model shows a crucial unsatisfactory aspect. In the process of formulating the model, there were two implicit assumptions. First, transport costs were evaluated presuming that there was only one type of household. In other words, there was no explicit consideration of a class-specific behaviour in the lower level, even though households were divided into several classes at the upper level. This caused difficulties in representing the different behaviour of households in the transport decision. The difficulties meant that inter-class interactions would not be considered. The other assumption was the use of a single criterion for classifying households. In many cases, the single index would not classify households into distinct sub-groups. A more realistic representation can be implemented using multidimensional measures. The two assumptions can be mitigated by incorporating a multiclass framework. The extension will be considered in the next section.

3.4. A multiclass bid-rent network equilibrium model

3.4.1. Introduction

As discussed at the end of the previous section, the lower level in the bid-rent network equilibrium model unsatisfactorily represents the diverse transport behaviour of households. The restriction results from the implicit assumption in which households show an identical decision-making process in the context of the transport behaviour. This section mitigates the limitation incorporating a multiclass framework, which explores a multiclass bid-rent network equilibrium model. The multiclass model integrates the existing framework of non-locators with a multiclass methodology that addresses interclass transport interactions. The combination produces an extended model of the multiclass transport decision. The multiclass transport decision is combined with the locational decision of households, which establishes the multiclass bid-rent network equilibrium model. The resulting formulation is understood as a seamless oligopolistic Cournot game for occupying residential locations.

3.4.2. Multiclass interactions

The behaviour of households is too diverse to assume that all households share a common decision-making rule concerning transport choices. In other words, the assumption that all users on the same network show an identical decision-making process for the transport decision is over-simplified. The assumption is far from a realistic network representation. This suggests a need for an alternative methodology that can mitigate the restriction of the identical behaviour. The approach adopted in this study to overcome the limitation is a framework of multiple user classes. The multiclass approach represents the behaviour of a number of different users allowing interactions within a user class as well as between classes (Dafermos, 1971; Van Vliet et al., 1986). In particular, the inter-class interactions should be considered for a realistic network specification. The interpretation of the framework for this study is that households are divided into several distinct classes, each of which has an individual strategy. The strategy contributes to its own and to the other classes' transport decision. In other words, a class has a specific transport cost function. The function contributes to its own and to the other classes' transport impedance.

Table 3-2 Example of criteria for the classification of user classes

Studies	Criteria	Stratification (e.g.)
Van Vliet et al. (1986)	Vehicle type	Car / Heavy vehicle
	Route choice criterion	Distance / Travel time
	Network restriction	Ordinary / Bus-only lane
Ran and Boyce (1994)	Income	High / Middle / Low
	Age	High / Middle / Low
	Route Diversion Willingness	One / A few / Frequently
	Driving Behaviour	Cautious / Rushed / Ruthless

A diverse combination of rules for the classification of user groups can be considered. Table 3-2 shows an example of the rules for specifying household classes. Van Vliet and others (1986) suggested three criteria of vehicle type, route choice criterion, and network restriction for the stratification of travellers. Ran and Boyce (1994, p. 249) proposed income, age, route diversion willingness, and driving behaviour as the criteria. In many studies, a mode-specific factor has been widely used for the grouping because transport modes would be the most distinguishable index for classifying the behaviour of network users regarding transport choices (Bliemer, 2001).

A desirable criterion would be the factor that can reasonably represent the nature of transport and the location of activities classifying households into distinct groups. In this study, two criteria are considered, namely the level of income and modes to travel. The factor income is regarded as a representative component to address the socio-economic characteristics of households. The bidding behaviour of households would be differentiated according to the level of income. In addition, the degree of income would produce a considerable gap between the perception from households to locational attractiveness and transport costs. The second criterion, transport modes, represents a physical restriction in the transport decision of households. The factor can involve various vehicle types and network restrictions such as bus-only lanes. Either income or mode can be used to classify households, which is referred to as a single dimensional criterion, namely C_y or C_m . Both factors, if necessary, can be integrated to identify user classes, which are referred to as a bi-dimensional criterion $C_{y \times m}$.

3.4.3. Transport decision: the problem of non-locators

This section presents an extension of the transport decision in the single class bid-rent network equilibrium model to that of the multiclass counterpart incorporating a multiclass framework. The resulting formulation represents the multiclass behaviour of non-locators in the transport decision. This behaviour is interpreted as analogous to an oligopolistic Cournot game. The discussion of the extension is heavily dependent on the studies of Dafermos (1971, 1972) and Netter (1972).

The single class transport network, which was suggested in the previous section, consisted of the directed mathematical network G , the set of travel demands Q , and the set of transport cost functions C ; they were represented as a scalar. In contrast, a multiclass network is represented by an m -dimensional vector of single class networks. The conceptual diagram for the multiclass network is shown in Figure 3-3.

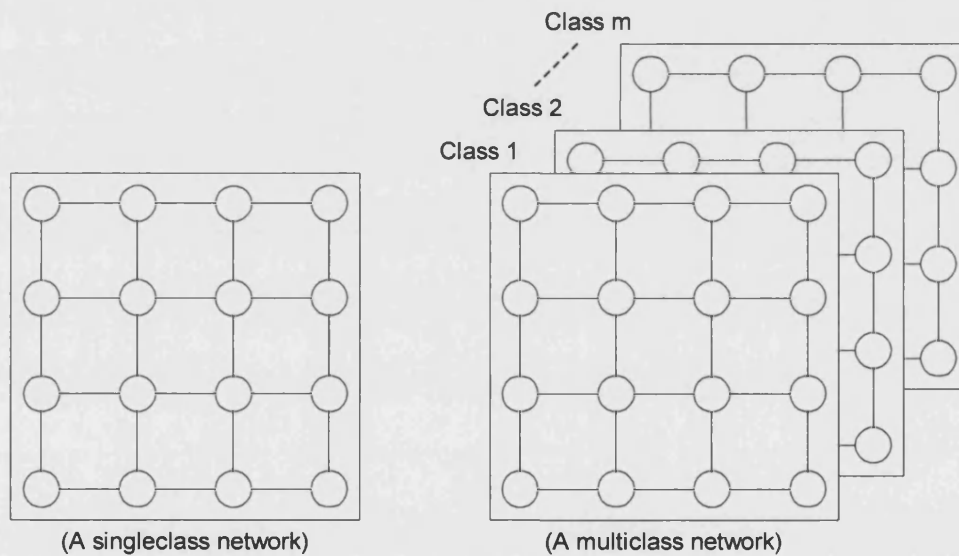


Figure 3-3 A conceptual diagram for the multiclass network

Let $\bar{G} = [\bar{N}, \bar{A}; M]$ denote a m -dimensional mathematical network where \bar{N} represents a set of m -dimensional nodes, \bar{A} a set of m -dimensional links, and M a set of user classes. Every class m , $\forall m \in M$ is associated with its own individual copy of a network, see Figure 3-3. Thus, the network \bar{G} is the union of m identical single class networks. Let $\bar{R}, \bar{R} \subseteq \bar{N}$ denote a set of m -dimensional origin nodes, $\bar{S}, \bar{S} \subseteq \bar{N}$

a set of m -dimensional destination nodes, and \bar{P}^{rs} a set of m -dimensional paths between an origin-destination pair r and s . There are nonnegative demands for travel $\bar{Q} = [q_m^{rs} : \forall m \in M, \forall r \in \bar{R}, \forall s \in \bar{S}]$ in the network \bar{G} . Like the single class model, the demands include both the locating and the non-locating households. It is also noted that the spatial interactions are represented by the unit of household members.

Let v_a^m be the flow of a class m on a link a , $\forall a \in \bar{A}$. The multiclass link volumes are

$$\bar{v}_a = (\dots, v_a^m, \dots), \forall a \quad (3.34)$$

Let f_{pm}^{rs} be the flow of a class m on a path p , $\forall p \in \bar{P}^{rs}$ that connects an origin-destination pair $\{r, s\}$. The vector of multiclass path flows is given by

$$\bar{f}_p^{rs} = (\dots, f_{pm}^{rs}, \dots), \forall p, r, s \quad (3.35)$$

The flow of a class m on a link a is equal to the sum of the flows of the class on the paths that contain the link.

$$v_a^m = \sum_p f_{pm}^{rs} \cdot \delta_{ap}^{rs}, \forall a, m \quad (3.36)$$

where δ_{ap}^{rs} is an indicator, namely $\delta_{ap}^{rs} = 1$, if a link a is on a path p , and 0 otherwise.

The spatial interaction flow of a class m for an origin-destination pair r and s is equal to the sum of the flows of the class m on paths that connects the OD pair.

$$q_m^{rs} = \sum_p f_{pm}^{rs}, \forall m, r, s \quad (3.37)$$

A nonnegative path flow that satisfies the conservation equation between the path flow and the spatial interaction demand is feasible and a *feasible flow pattern* $\bar{\Omega}$ is defined by a family of individual feasible path flows:

$$\bar{\Omega} = [f_{pm}^{rs} : \forall p \in \bar{P}^{rs}, \forall m \in M, \forall r \in \bar{R}, \forall s \in \bar{S}] \quad (3.38)$$

The travel demands \bar{Q} are associated with transport costs. Let t_a^m be the transport cost of a user class m on a link a . The m -dimensional vector of link cost functions is

$$\bar{t}_a = (t_a^1, \dots, t_a^m), \forall a \quad (3.39)$$

It is assumed that the transport cost on a link a depends on the flow of the link a :

$$\bar{t}_a = \bar{t}_a(\bar{v}_a), \forall a \quad (3.40)$$

This study considers complete inter-class interactions on a link a . This suggests that the transport cost function can be rewritten as follows:

$$\begin{aligned} t_a^1 &= t_a^1(v_a^1, \dots, v_a^m), \forall a \\ &\vdots \\ t_a^m &= t_a^m(v_a^1, \dots, v_a^m), \forall a \end{aligned} \quad (3.41)$$

where the re-specified link cost function clearly shows that each class has an individual transport cost function and the cost function contributes to its own and the other classes' transport impedance, which is compatible with the definition of the inter-class interaction in this study.

The functions of the transport impedance are set, $\bar{C} = [t_a^m(\bar{v}_a) : \forall m \subset M, \forall a \in \bar{A}]$. Finally, the multiclass network is defined as combining the multiclass directed mathematical network \bar{G} , the multiclass travel demand \bar{Q} , and the multiclass transport cost function \bar{C} , namely $\bar{N} = [\bar{G}, \bar{Q}, \bar{C}]$.

The cost function in the multiclass network \bar{N} requires a significant number of parameters that are specific to the decision-making rule for each user class. In the Dafermos example (1972), the number of parameters was 35 in the only two-class seven-link network. It would be almost impossible to specify all necessary parameters in a large-scale network. Alternatively, the normalising approach can be used (Van Vliet et al., 1986; Lam and Huang, 1992a; Lam and Huang, 1992b).

$$t_a^m = t_a^m(v_a^1, \dots, v_a^m) = w^m \cdot t_a(v_a), \forall a, m \quad (3.42)$$

where w^m is a weighting factor for a user class m . The factor w^m is normally associated with the value of travel time for the user class m .

The normalised transport cost function shows that a class m has its own link cost function and the function contributes its own and the other classes' transport impedance, which is in line with the assumption of the multiclass formulation.

In the normalising approach, the path cost of a class between $\{r, s\}$ is given by

$$\begin{aligned} c_{pm}^{rs} &= \sum_a t_a^m (v_a^1, \dots, v_a^m) \cdot \delta_{ap}^{rs}, \forall m, p, r, s \\ &= w^m \cdot \sum_a t_a (v_a) \cdot \delta_{ap}^{rs} \end{aligned} \quad (3.43)$$

where the equation shows again that path costs are determined by a joint function of the load of its own class and those of the remaining classes. In other words, the cost c_{pm}^{rs} is dependent on the flow of its own class as well as those of the other classes.

In this study, travellers have been assumed to explicitly consider the two network performance indicators of transport impedance and locational attractiveness in the transport decision. In other words, trip-makers have been supposed to seek a location that satisfies the composite goal of minimising travel time and maximising locational attraction. The net interaction impedance has been provided to represent the explicit consideration of the composite speculation. The multiclass version is provided:

$$|c_{pm}^{rs} - \bar{\varphi}_m^r|, \forall m \quad (3.44)$$

where $\bar{\varphi}_m^r$ is class specific locational attractiveness of a residence r . As noted in the single class model, the value is represented in the unit of transport cost.

An extension of the equilibrium condition of the single class model to that of the multiclass counterpart is straightforward.

Definition 8. A feasible flow f_{pm}^{rs} is a user optimal if, for each OD pair $\{r, s\}$ and class m , there is a quantity η_m^{rs} that satisfies the following properties:*

$$|c_{pm}^{rs} - \bar{\varphi}_m^r| \begin{cases} = \eta_m^{rs} & \text{if } f_{pm}^{rs*} > 0, \forall m, r, s \\ \geq \eta_m^{rs} & \text{if } f_{pm}^{rs*} = 0, \forall m, r, s \end{cases} \quad (3.45)$$

where η_m^{rs} is a minimal net interaction impedance for a user class m and f_{pm}^{rs} is an equilibrium path flow of a class m .*

The multiclass equilibrium condition suggests that all locations chosen by each class m have the same net interaction impedance. The impedance is at least the same as those of the other alternatives that are not chosen. This condition is represented by minimising the difference between class specific transport costs and locational benefits, namely $|c_{pm}^{rs} - \bar{\varphi}_m^r|$, $\forall m \in M$. A formulation using a variational inequality is proposed to represent the minimisation. Preliminarily, a path cost function is extended incorporating a multiclass component.

$$\tilde{c}_{pm}^{rs} = \tilde{c}_{pm}^{rs}(f_{pm}^{rs}, \bar{\varphi}_m^r) = \sum_a t_a^m \cdot \delta_{ap}^{rs} - \bar{\varphi}_m^r, \forall m, p, r, s \quad (3.46)$$

where the path-based multiclass cost function $\tilde{c}_{pm}^{rs}(\cdot)$ is flow-dependent because the link cost t_a^m has been assumed to be flow-dependent.

The link cost function in the single class bid-rent network equilibrium model was also flow-dependent. Thus, it is useful to clarify the characteristics of the link cost functions between the two models. Both functions are a separable type, which means that the two cost functions consider no interactions between links. In other words, the cost of a link solely depends on the volume of the link. The multiclass model allows the diverse combinations of class-specific flows on a link. Hence, the link cost depends on the vector of the link flows of user classes. This means that there are interactions between the classes on the network. These interactions might be symmetric. However, the symmetry condition is very restrictive because the condition requires that the impact of a class to the other classes on a link should be the same as those of the other classes. Thus, it is reasonable to assume that the link cost function is an asymmetric type, which this study adopts. When the cost function is asymmetric, there are no known equivalent minimisation programmes. A variational inequality can be used as an alternative formulation. A variational inequality formulation is more general than an optimisation programme because the framework can flexibly handle the asymmetric interaction between classes (Smith, 1979; Dafermos, 1980).

The minimisation of the net interaction impedance for the multiclass bid-rent network equilibrium model is formulated using a variational inequality, which finds $\{f_{pm}^{rs*}\}$,

$$\forall f_{pm}^{rs} \in \bar{\Omega} \text{ where } \bar{\Omega} = [f_{pm}^{rs} : \forall p \in \bar{P}^{rs}, \forall m \in M, \forall r \in \bar{R}, \forall s \in \bar{S}] :$$

$$\begin{aligned}
& \sum_m \sum_{rs} \sum_p \tilde{c}_{pm}^{rs} (f_{pm}^{rs*}, \bar{\varphi}_m^r) (f_{pm}^{rs} - f_{pm}^{rs*}) \geq 0 \\
& s.t. \quad \bar{\varphi}_m^r = E \left[\text{Max}_{H \in m} U_{H \in m}^{r\sigma}(\cdot) \right] \quad \forall m, r \\
& \quad \sum_p f_{pm}^{rs} = q_m^{rs} \quad \forall m, r, s \\
& \quad f_{pm}^{rs} \geq 0 \quad \forall m, p, r, s
\end{aligned} \tag{3.47}$$

The formulation represents the Nash equilibrium; no household class can save resources in terms of the net interaction impedance by deviating from the equilibrium flow $\{f_{pm}^{rs*}\}$. This suggests that no class has an incentive to alter its strategy to win the competition. The Nash equilibrium inferred addresses an oligopolistic competition of households. In other words, the formulation satisfactorily describes the class-to-class game. In fact, there was an inconsistency in terms of a game theoretical interpretation in the single class model. The overall formulation of the single class model has represented the oligopolistic Cournot game. The lower level model, however, has suggested the many-to-many competition even though the group-to-group game has been considered in the upper level. The inconsistency is successfully resolved in the multiclass model. The formulation clearly represents the class-to-class competition, namely an oligopolistic game. Furthermore, the Nash equilibrium can be systematically defined. Hence, the multiclass bid-rent network equilibrium model can be regarded as a seamless analogy to the oligopolistic Cournot game.

Braess and Koch (1979) showed that multiclass models have at least one solution when the link cost function is continuous and monotone. The general condition for the existence of solutions has been established by Smith (1979, 1981, 1983a). Here, a compact version analogous to the case of the single class model is given. The variational inequality has at least one solution provided that the set of multiclass feasible path flows is non-empty (see Bell and Iida, 1997, pp. 86-89).

The uniqueness condition of a solution for the total link flow case has been suggested (Van Vliet et al., 1986; Nagurney, 2000; Nagurney and Dong, 2002a; Nagurney and Dong, 2002b). In this theorem, the total link flow pattern $\bar{v}_a, \forall a$ induced by the weighted sum of the link flow patterns of each user class $\bar{v}_a^m, \forall a, m$ is unique if the

link cost function is separable and strictly monotone. However, this condition is self-contradictory in the sense that the theorem assumes a class-independent cost function even though the definition of the multiclass problem requires interactions between user classes, namely a class-dependent cost function (Watling, 1996).

Dafermos (1980) tried to establish the general uniqueness condition assuming strongly monotone link cost functions. The strong monotony condition is satisfied if and only if the Jacobian matrix is positive definite (Dafermos, 1980). This condition implies that the inter-class interaction is weak. In other words, the dominant factor that determines the link cost of each class is its own flow, although the transport impedance depends on the entire flows of all the classes. However, this is not always applicable in most situations. Thus, the multiclass model could have multiple solutions in terms of the vector of equilibrium link flows; inevitably, the vector of equilibrium path flows as well. As noted in the single class model, this property does not mean that the multiclass net interaction impedance cannot be defined. An appropriate solution algorithm could offer an equilibrium flow patterns; this issue is presented in Chapter Four, Solution Algorithm.

In summary, this section has discussed the transport decision of non-locators incorporating a multiclass framework. The resulting formulation satisfactorily represents diverse class specific behaviour as well as the inter-class interaction. The formulation has been understood as equivalent to an oligopolistic Cournot game in the transport decision. This satisfies the consistency in terms of the game theoretical interpretation for the formulation of the bid-rent network equilibrium model.

3.4.4. Formulation

In the previous two sections, the conceptual framework for representing the inter-class interaction and the incorporation of the multiclass approach to the transport decision were considered. In this section, a resulting formulation is provided combining the improved transport decision and the locational decision of households, namely the multiclass bid-rent network equilibrium model. Let $Pr^{rs}(\cdot)$ be the share of the total number of locating households between a residential location r and a primary activity location s , $\forall r \in R, \forall s \in S$; $0 \leq Pr^{rs}(\cdot) \leq 1$ and $\sum_r Pr^{rs}(\cdot) = 1$. Let $U_{H \in m}^{r\sigma}$ be the utility function of a household H , $U_{H \in m}^{r\sigma} = U_{H \in m}^{r\sigma}(\varphi^r, u_H^{r\sigma}; \beta)$. Let \bar{D}_m^s be the class-specific total number of household members attracted at an economic activity location.

$$\begin{aligned}
 Pr^{rs}(m | z) &= Pr(U_{H \in m}^{r\sigma} \geq U_{H' \in m'}^{r\sigma}) & \forall m \subset M \\
 s.t. \quad \varphi^r &= \bar{\varphi}_m^r / y_H & \forall r \\
 u_H^{r\sigma} &= \sum_{h \in H} \tilde{c}_{pm}^{rs}(h; f_{pm}^{rs*}, \bar{\varphi}^r) & \forall H, r, s \\
 \sum_h \sum_r h_m^{rs} &= \bar{D}_m^s & \forall m, s \\
 f_{pm}^{rs} &\geq 0, \varphi^r \geq 0, u_H^{r\sigma} \geq 0, z_i \geq 0 & \forall H, i, m, p, r, \sigma
 \end{aligned} \tag{3.48}$$

where $u_H^{r\sigma}$ is obtained by finding the equilibrium flow $\{f_{pm}^{rs*}\}$, $\forall f_{pm}^{rs*} \in \bar{\Omega}$ using an origin-destination matrix that is calculated as follows: $\{q_m^{rs}\} = \{\tilde{q}_m^{rs} + Pr^{rs}(\cdot) \times \bar{D}_m^s\}$

$$\begin{aligned}
 \sum_m \sum_{rs} \sum_p \tilde{c}_{pm}^{rs}(f_{pm}^{rs*}, \bar{\varphi}_m^r)(f_{pm}^{rs} - f_{pm}^{rs*}) &\geq 0 \\
 s.t. \quad \bar{\varphi}_m^r &= E(\text{Max} U_{H \in m}^{r\sigma}) & \forall m, r \\
 \sum_p f_{pm}^{rs} &= q_m^{rs} & \forall m, r, s \\
 f_{pm}^{rs} &\geq 0 & \forall m, p, r, s
 \end{aligned} \tag{3.49}$$

where $\bar{\Omega} = [f_{pm}^{rs} : \forall p \in \bar{P}^{rs}, \forall m \subset M, \forall r \in \bar{R}, \forall s \in \bar{S}]$ is the set of feasible solutions.

The hedonic-based random bid-rent model in the upper level shows a slight difference from the formulation of the single class bid-rent network equilibrium model. The equilibrium path flow in the multiclass model has been represented for the class specific solution $\{f_{pm}^{rs*}\}$ rather than the overall equilibrium solution $\{f_p^{rs*}\}$ as it does in the single class model. This difference suggests that the multiclass model considers

the class specific transport impedance when the model represents the behaviour of locators. The lower level has been significantly improved. The changes have been invoked by the incorporation of the multiclass approach. The embodiment makes the model address the interaction between household classes in the transport decision. The overall changes in terms of the multiclass framework have been smoothly combined in the bi-level formulation. This suggests a consistency with the single class model in representing the nature of the relationship between transport and the location of activities. It is reminded that the nature has involved the three criteria of the unique characteristics of locations, the decision-making process of households, and the interaction between transport and land-use. The systematic connection between the components produces the endogenous network performance indices of transport impedance and locational attractiveness. The endogenous solutions have been regarded as a unique feature of this model. A further important point that should be noted in the formulation is a game theoretical interpretation. Even though the single class model has discussed the behaviour of households in the context of the oligopolistic Cournot game, the lower level has represented the n-player competition implicitly assuming that the behaviour of travellers would be identical in the transport decision. The incorporation of the multiclass approach in the multiclass bid-rent network equilibrium model successfully describes the competition of the oligopoly in the lower level. This suggests a seamless oligopolistic Cournot representation of the multiclass model in each level as well as in the overall formulation.

3.4.5. Discussion

In this section, the extension of the bid-rent network equilibrium model to the multiclass perspective has been discussed. The multiclass approach was understood as a useful framework for representing the interaction between household classes in the transport decision. The framework was embodied in the model of non-locators. The resulting formulation suggested an equivalent oligopolistic Cournot game.

The multiclass framework incorporated in this section dealt with the interaction between user classes on the same link only. However, the transport cost on a link might be affected by the flows on other links as well. This means that transport costs might be a function of entire network factors involving different classes and overall network circumstances (Nagurney and Dong, 2002a). An extension of the model incorporating the inter-class and inter-link interactions is straightforward. A slight modification to the link cost function could address the dual interactions.

Let \bar{v} be the matrix of class-link flows

$$\bar{v} = \begin{pmatrix} v_1^1, \dots, v_1^m \\ \dots\dots\dots \\ v_a^1, \dots, v_a^m \end{pmatrix} \quad (3.50)$$

The transport cost on a link a of a class m is given by

$$t_a^m = t_a^m(\bar{v}), \quad \forall a, m \quad (3.51)$$

The subsequent derivation of the model with the dual interactions is identical to that of the single-interaction multiclass model. In a theoretical context, the extension would not be a difficult quest. A slight modification to the link cost function would enable the model to represent the interactions. The improvement, however, would not be an easy task practically. This is because the extension requires a comprehensive reference that specifies the relationship between user classes and network components. This inevitably accompanies a site-specific survey. In particular, when a large-sized network is considered, this would be merely theoretically applicable.

3.5. Conclusion

The bid-rent network equilibrium model has been developed in this chapter. The model was designed to represent the three nature of the relationship between transport and the location of activities. First of all, the hedonic interpretation of the model reasonably recognised the characteristics of heterogeneity and indivisibility of locations. Secondly, the composite decision-making process of households, which meant interactions between decisions within a household and those between households, was represented using a game theoretical framework, accompanied by a bidding mechanism. Finally, a mutual adjustment process addressed the interaction between transport and land-use. The process produced the endogenous network performance indicators of transport impedance and locational attractiveness.

The process of developing the model was considered in the context of competition for locations. The two household groups, which were locators and non-locators, were identified. The behaviour of locators was considered using the hedonic-based random bid-rent model. The minimisation of the net interaction impedance was provided for representing the behaviour of non-locators. The two components were embodied in a bi-level programme. The formulation was interpreted as equivalent to an oligopolistic Cournot game. The Nash equilibrium for this game was defined. The bid-rent network equilibrium model was improved incorporating a multiclass framework. The extended formulation systematically represented inter-class interactions. The resulting model was understood as a seamless oligopolistic competition of the Cournot game.

In conclusion, the bid-rent network equilibrium model developed in this chapter satisfactorily represents the three structural nature of the relationship between transport and the location of activities. The systematic connection between the theories for the conceptual basis and the methodologies for the practical modelling would mean that the model could be an alternative framework for representing this problem. There is little doubt that the bid-rent network equilibrium model is useful for investigating the interaction between transport and the location of activities.

4. Solution Algorithm

4.1. Introduction

This chapter proposes an appropriate solution algorithm for the bid-rent network equilibrium model. The model was formulated using a bi-level programme. Solving the bi-level model, however, is not a simple task. The difficulty results from the intrinsic characteristics of the programme (Friesz et al., 1990). First, the programme is inherently non-convex because of the implicit functional relationship between the decision variables of the upper and lower levels. The non-convexity means that finding the global solution would be difficult. Secondly, the model requires an intensive computational process. Each level of the programme demands an incomplete (or complete) solution at the other level when the level updates a solution. This intermediate solution is required many times until convergence is achieved, which causes a huge computational effort. Thus, the solution-finding process for the optimisation problem has long been considered an important yet complicated and challenging quest (Yang and Bell, 2001). The two unhelpful characteristics are also found in the proposed formulation of the bid-rent network equilibrium model. Thus, the goal of the solution algorithm for the model, which will be suggested in this chapter, is to produce a good solution rather than a unique answer.

In the first section, a review on the existing solution algorithms for bi-level mathematical programmes is considered. The algorithms are classified into deterministic and stochastic types. The deterministic algorithm is subdivided into responsive and iterative algorithms. In the subsequent section, a solution algorithm for the bid-rent network equilibrium model is proposed. This study adopts the iterative algorithm to find a mutually consistent solution that represents the Nash equilibrium; note that the proposed model intends to address the Nash equilibrium. Specifically, a heuristic routine combines a path-based algorithm for solving the lower level and the Newton-Raphson procedure for estimating parameters at the upper level. In the following section, simple numerical examples that illustrate the operation of the bid-rent network equilibrium model are suggested. Some results from the solution algorithm are discussed. In the final section, concluding remarks are presented.

4.2. Existing algorithms

4.2.1. Introduction

As discussed in the introductory section of this chapter, bi-level mathematical programmes have two unhelpful features in finding a solution: first, the non-convex structure of the programmes suggests difficulties in identifying a global solution; secondly, the programme requires a significant computational process. These two properties have motivated existing approaches to investigate heuristic algorithms that normally use an alternative improvement strategy (Yang and Bell, 1998). In other words, the heuristic algorithms calculate a solution updating the problem of the upper and lower levels alternately. Thus, the process consists of a successive alternation between the levels exchanging solutions with each other.

Heuristic algorithms are designed to find an acceptable answer for typical problems in a reasonable computation time (Rutenbar, 1989). The heuristics, however, have no guarantee producing an optimum solution. Therefore, the goal of the heuristic algorithms focuses on identifying a reasonable solution rather than a unique one. This suggests that the algorithms attempt to perturb some existing sub-optimal solutions in the direction of a better solution. The process continues until no further improvements are obtained, at which point the procedure terminates (Boyce and Mattsson, 1999).

In this section, existing solution-finding mechanisms for bi-level mathematical programmes are examined. The algorithms are classified into deterministic and stochastic types. While the deterministic algorithm produces the same solution every time the algorithm runs, the stochastic type generates different answers. The deterministic algorithm can be sub-grouped into iterative and responsive types in terms of a game theoretical context. When an algorithm focuses on finding a mutually consistent solution, which suggests the Nash equilibrium, the algorithm is referred to as an iterative type. In contrast, a responsive algorithm, which suggests the Stackelberg equilibrium, highlights a leader-follower structure. Finally, the existing types of algorithms are compared. The comparison concludes that no solution algorithm guarantees producing a global solution in an efficient computation process.

4.2.2. Iterative algorithms

Iterative algorithms produce a mutually consistent solution, which is equivalent to the Nash equilibrium. As discussed in the chapter for the model development, the Nash equilibrium could be defined in every n-player non-cooperative game. A particularly interesting type of n-player non-cooperative games in transport studies is the Cournot game. The game is originally a framework to analyse duopoly or oligopoly. This means that small numbers of gamers who have conflicts of interests are considered. Each gamer is assumed to maximise his or her profits simultaneously. In this process, an individual gamer is supposed to follow the best strategy in response to every other player since no player has a dominant situation. At equilibrium, no player has an incentive to deviate from the equilibrium because their strategy is the best response to their belief about the strategies of the other players.

$$Y(S_i^*, S_{-i}^*) \geq Y(S_i, S_{-i}^*) \quad (4.1)$$

where $Y(\cdot)$ is the payoff function of players;

S_i^* is the optimal strategy of a player i ; and

S_{-i}^* is the optimal strategy profile of all players except i , namely

$$S_{-i}^* = (\dots, S_{i-1}^*, S_{i+1}^*, \dots).$$

The process of the iterative algorithm is analogous to the play rule of the Cournot game. The successive exchange of solutions between levels in the algorithm describes the rule of the best response of players in the game. The Nash equilibrium in the Cournot game is represented by a mutually consistent solution in the algorithm.

Implementations of the iterative algorithm are simple and straightforward. The algorithm solves the upper level problem while holding fixed the decision variables of the lower level, and then solves the lower level problem while holding fixed the decision variables of the upper level. This process continues until the problem converges in equilibrium (Bell and Iida, 1997). The general procedure of the iterative algorithm can be summarised as follows:

Step 0. Initialise the upper level solution $\{\mathbf{x}^0\}$; set $k=0$

Step 1. Find the lower level solution $\{y^k\}$ with $\{x^k\}$

Step 2. Determine the upper level solution $\{x^{k+1}\}$ with $\{y^k\}$

Step 3. If there is an improvement of a solution, then set $k=k+1$ and go to 'Step 1'; otherwise, stop the procedure.

The iterative algorithm has been applied to various research areas. In particular, the algorithm has been popularly used in studies for investigations into the relationship between signal settings and network flows. In this question, the route choice of drivers is regarded as a function of network design parameters. For example, Allsop (1974) emphasised that a realistic approach to a signal setting was to find a design parameter that considered the network behaviour of drivers. This was represented by means of combining a problem of a traffic signal setting with a travel demand model. The ultimate purpose of the research was to find the influence of signal settings on traffic patterns. In other words, the model investigated how traffic controls influenced travel costs and traffic flows. For a solution procedure, the iterative algorithm was suggested. The process calculated alternately between the signal setting and the travel demand until the problem converged in equilibrium. This result was applied to a signal-controlled network for demonstrating different signal timings would induce different routings (Allsop and Charlesworth, 1977). The algorithm produced two mutually consistent solutions which could be interpreted as network users had no incentive to change their route, and a planner had no reason to alter the design parameter; these were consistent with the Nash equilibrium. However, since the system designer was expected to act, taking into account the responses of the network users, the solutions found could not satisfactorily represent the behaviour of the two agents. This was because the solution of one problem only took into account the solution of the other in the previous iteration. Furthermore, the two different solutions signified the difficulty of the iterative algorithm in finding a global solution.

A similar procedure was used in SATURN (the Simulation and Assignment of Traffic in Urban Road Network) model (Bolland et al., 1979; Hall et al., 1980). SATURN, which had two phases of simulation and assignment steps, was a simulation tool for traffic management schemes. The two phases were connected by an iterative loop; the simulation phase calculated the parameters of the flow-delay curve given the traffic

pattern, and then the assignment phase recalculated route flows and costs using the flow-delay function which was given by the simulation phase. These iterations continued until an equilibrium was obtained. The ENETS (*Equilibrium NETWORK traffic Signal setting*) model (Cantarella et al., 1991) was another similar example of the application of the iterative algorithm in the problem of the combination between the signal setting and the traffic routing. In ENETS, the model of the traffic signal setting consisted of two steps: first, the capacity maximisation of green timing and scheduling at a single junction; second, the delay minimisation of signal coordination. ENETS highlighted the effect between changes in the control strategy and the redistribution of flows in the entire network.

Other interesting applications of the iterative algorithm can be found in the literature. For example, the algorithm was used to calibrate the parameters of road capacity function in Korea (Suh et al., 1990). Suh and others claimed that the widely used US BPR (Bureau of Public Roads) parameters might not be appropriate in other areas because of their unique transport environments. A bi-level programming model was suggested to calibrate the parameters suitable for Korean highway circumstances. The model combined a least square measurement between the observed and estimated link flows with a user optimal assignment model. Boyce and Mattsson (1999) formulated a bi-level framework of the housing supply, which was one of the sub models of IMREL (*Integrated Model of Residential and Employment Location*). In the upper level, the problem was to find housing supply patterns. This was represented by maximising the welfare of workers. In the lower level, the aggregate behaviour of employee was investigated given the locational pattern of jobs and housing types. The usual iterative procedure was proposed to calculate a solution. An interesting variation of the iterative algorithm is the *Equilibrium Decomposed Optimisation* (EDO) technique (Suwansirikul et al., 1987). The approach decomposes an objective function at each link, and then simultaneously solves the function using a one-dimensional search routine such as Fibonacci, Golden Section, and Bisection. The usual iterative procedure between the upper and lower levels is used to update the solution.

4.2.3. Responsive algorithms

Responsive algorithms suggest a solution that would be equivalent to the Stackelberg equilibrium which is one of the Nash type equilibriums; see Appendix I. It would be useful to distinguish the Stackelberg game from the Cournot game. There are similarities and dissimilarities between the two games. Both games deal with small numbers of gamers, namely a duopoly and an oligopoly. In these games, no player has a dominant strategy. The non-dominance means that the framework of the best-response analysis can be applied to investigate the behaviour of decision-makers in these games. The main difference between the two games is observed in the order of gamers' actions. While the Cournot game assumes that all players act simultaneously, the Stackelberg game assumes a sequential decision-making process. In other words, in the Stackelberg game, one player acts before the others. The first mover is referred to as a leader. The others are denoted as followers. For this reason, the Stackelberg game is referred to as a leader-follower game. The leader begins the game by announcing its decision. The followers execute their policies after the decision of the first mover. The leader tries to maximise its profit taking the reasonable reaction of the followers into account. In contrast, the followers simply react to the leader's choice, namely the best response to the leader's decision. Therefore, the leader has an advantage, which is referred to as the *first-mover advantage*. Mathematically, the equilibrium state of the Stackelberg game can be expressed as follows:

$$\begin{aligned} \Upsilon(S_i^*, \Phi(S_{-i}^* | S_i^*)) &\geq \Upsilon(S_i, \Phi(S_{-i}^* | S_i)) \\ \text{and} \\ \Upsilon(S_i^*, \Phi(S_{-i}^* | S_i^*)) &\geq \Upsilon(S_i^*, \Phi(S_{-i} | S_i^*)) \end{aligned} \quad (4.2)$$

where i is the leader and $\Phi(\cdot)$ is the response function of the followers.

As stated above, the responsive algorithm produces an analogous solution to the Stackelberg equilibrium. This suggests that the algorithm would represent the leader-follower structure. In this framework, a traffic agency is normally set to be the leader and the group of network users are regarded as the followers. The algorithm explicitly represents the two distinct characteristics of the Stackelberg game. First, the first-mover advantage is considered by solving the problem of the followers given the solution to the problem of the leader. Secondly, the reaction of the followers is

represented in the algorithm calculating the problem of the leader without fixing the decision variable of the followers. The first characteristic is also found in the iterative algorithm, but the second is a unique feature of this type of algorithm. A step for evaluating the reaction factor has been introduced to represent the second characteristic in the responsive algorithm.

Step 0. Initialise the upper level solution $\{\mathbf{x}^0\}$; set $k=0$.

Step 1. Find the lower level solution $\{\mathbf{y}^k\}$ with $\{\mathbf{x}^k\}$

Step 2. Calculate the reaction factor $\{\Phi^k\}$ at $\{\mathbf{y}^k\}$

Step 3. Determine the upper level solution $\{\mathbf{x}^{k+1}\}$ with $\{\Phi^k\}$

Step 4. If there is an improvement in the solution, then set $k=k+1$ and go to 'Step 1'; otherwise, terminate the procedure.

This group of algorithms can be divided in terms of the way the reaction factor is evaluated. Three procedures are found in the literature, namely the *Link Usage Proportion-Based (LUPB)* and the *Sensitivity Analysis-Based (SAB)* algorithms.

The LUPB algorithm explicitly considers the reaction factor. The algorithm evaluates the reaction factor using a path flow solution to the lower level problem.

$$\Phi_a^{rs} = \frac{\sum_p f_p^{rs} \cdot \delta_{ap}^{rs}}{x^{rs}}, \quad \forall a \in A, \forall r \in R, \forall s \in S \quad (4.3)$$

where Φ_a^{rs} is the link usage proportion on a link a of an OD pair r and s ;

f_p^{rs} is the flow on a path p connecting an OD pair r and s ;

x^{rs} is the decision variable of the upper level; and

δ_{ap}^{rs} is an indicator; $\delta_{ap}^{rs} = 1$, if a link a is on a path p , and 0 otherwise.

The reaction factor is used to predict the changes in an equilibrium flow, which is interpreted as the reaction of the followers. This suggests that the LUPB algorithm would satisfactorily capture the leader-follower structure. The equilibrium flow, however, is not uniquely determined because a path-flow solution is not normally

unique. The algorithm has been applied to some areas: a design of ramp metering (Yang et al., 1994) and OD matrix estimation (Yang et al., 1992; Yang, 1995).

The SAB algorithm uses a sensitivity analysis in evaluating the reaction factor. The sensitivity analysis is a way of calculating a new approximate solution by means of parameter perturbations at a current solution. The perturbations may occur simultaneously across the parameters. Thus, the information on the sensitivity analysis can be used to test the stability of a solution as well as the degree of sensitivity of a particular factor. The direct application of the sensitivity analysis to the investigation of the network equilibrium is infeasible because the equilibrium path flow is not uniquely determined. Tobin and Friesz (1988) developed an alternative approach to overcome the problem, namely *the approach of the restricted problem*. The restricted approach would be described as follows: first, select a particular path flow solution, which is referred to as an extreme point; then, calculate the derivative of the extreme point with respect to the perturbation in parameters; lastly, transform the derivative of the path flow to that of the link flow. Tobin and Friesz (1988) proved that the derivative of the link flow is independent of the extreme point chosen. This means that the derivative information of the restricted problem can be regarded as equivalent to that of the original problem. This technique, however, is not feasible when the number of origin-destination pairs exceeds that of links, as most real networks do. This is because the Tobin and Friesz approach is based on complicated matrix calculation, and so the Hessian of the objective function with respect to the vector of path flows, which is essential for implementing the sensitivity analysis, is not invertible when the number of paths in the link-path incidence matrix exceeds the number of links. Therefore, this approach cannot be used in most practical-sized networks (Bell and Iida, 1997, pp. 95-100; Patriksson, 2004).

$$\Phi = \frac{\partial \mathbf{y}}{\partial \mathbf{x}} = [\mathbf{J}_\mathbf{x}]^{-1} \cdot [-\mathbf{J}_\epsilon] \quad (4.4)$$

where ϵ is the vector of the parameters of perturbation and
 \mathbf{J} is the Jacobian matrix of the lower level problem.

The SAB algorithm could represent changes in link flows more accurately than the LUPB algorithm because the SAB algorithm uses the derivative information in evaluating the reaction factor. There have been several applications of the sensitivity

analysis for developing solution algorithms outside transport studies. De Silva (1978) suggested the SAB algorithm for a model of US crude oil production. Friesz and others (1988) implemented the SAB algorithm for the spatial location problem. Since then, many transport applications of the SAB algorithm can be found in the literature: ramp metering on urban freeway networks (Yang et al., 1994; Yang and Yagar, 1994); a traffic signal setting (Yang and Yagar, 1995); an optimal congestion pricing (Yang and Lam, 1996; Yang and Bell, 1997); a reserve capacity approach to a signal control (Wong and Yang, 1997; Ziyou and Yifan, 2002); a trip table estimation (Yang, 1995); determining an optimal speed detector density (Chan and Lam, 2002); and a transit network design problem (Gao et al., 2003).

4.2.4. Stochastic algorithms

In the previous sections, deterministic algorithms were considered. The deterministic algorithms were classified in terms of a game theoretical context. When algorithms suggest a mutually consistent solution, the algorithms were denoted as the iterative algorithm. In contrast, the responsive algorithm represented the leader-follower structure of the Stackelberg game. This section deals with stochastic algorithms. Whilst the deterministic algorithms produce the same answers every time the algorithms run, the stochastic algorithms offer different solutions at each run. This is because the stochastic algorithms use a probabilistic transition rule when solutions are improved. Fundamentally, this group belongs to local search methods. The stochastic rule, however, allows the algorithms to search more points in the dominion of solutions than the deterministic counterparts. This characteristic may reduce the possibility of being trapped in local optima, which may increase the possibility of finding the global optimum. In this section, two representative stochastic algorithms are reviewed, namely the simulated annealing algorithm and the genetic algorithm.

The simulated annealing algorithm is a technique for solving combinatorial optimisation problems. The procedure of this approach is analogous to that of the statistical mechanics of annealing in solids (Kirkpatrick et al., 1983). In physics, a material that is at high temperature is a highly disordered state. This material is not very useful because the material can be easily defective. In contrast, as the temperature cools down, a material becomes an ordered state. If the cooling is done too quickly, the material is damaged or broken; whereas, a moderate control in cooling transforms a material into a solid body. Annealing is a procedure that coerces a material into a low energy state. In this process, a material is heated to a high-energy state that actively allows atomic rearrangements. Then, the material is carefully cooled until the material is converted into a solid body.

The procedure of the simulated annealing algorithm is similar to the process of the controlled cooling operation in physics. The algorithm seeks to transform a poorly disordered solution into a desirably ordered answer. The transformation is achieved by means of a perturbation towards better as well as worse configurations. The perturbation allows the algorithm to jump from the dominion in the direction of local

solutions, which potentially enables the algorithm to take a more promising path towards the global optimum. The procedure can be summarised as follows:

Step 0. Initialise a solution $\{\mathbf{x}^0, \mathbf{y}^0\}$; set $k=0$.

Step 1. Simulated annealing.

Step 1.1. Find thermal equilibrium.

Step 1.2. Cooling schedule.

Step 2. If a solution satisfies stopping criteria, then terminate the procedure; otherwise, set $k=k+1$ and go to 'Step 1'.

In finding thermal equilibrium, the Metropolis rule (1953) is usually adopted. The procedure generates a random perturbation near to a current temperature that represents a solution. The perturbation enables a particle to move towards a new location in a feasible region. Then, the step evaluates the resulting changes in energy. If the energy decreases, the new configuration is accepted as a new starting point for the next move. If the energy increases, the Boltzman distribution is applied. The Boltzman distribution would make the algorithm move towards a worse solution, which might enable the procedure to jump towards the global optimum. Finally, the algorithm checks whether the energy satisfies the thermal equilibrium. In practice, where few improvements across several successive temperatures are observed, the point is regarded as the thermal equilibrium. The process is summarised as follows:

For $k=1$ to K

/ generate a random perturbation */*

$$\mathbf{x}^k = \mathbf{x}^{k-1} + \Delta \mathbf{x}$$

If $|\mathbf{x}^k - \mathbf{x}^{k-1}| < 0$ then

/ accept a better solution */*

$$T^{new} = \mathbf{x}^k$$

ElseIf $|\mathbf{x}^k - \mathbf{x}^{k-1}| \geq 0$ then

/ apply the Boltzman distribution */*

/ calculate an acceptance probability */*

$$P = \exp(-|\mathbf{x}^k - \mathbf{x}^{k-1}| / T)$$

```

/* generate a random value R in [0,1] */
If P > R then
    /* accept a worse solution */
     $T^{new} = x^k$ 
Else
    /* no improvement in temperature */
     $T^{new} = T^{old}$ 
End If
End If
Next

```

In the cooling schedule step, the algorithm moderates the acceptance of worse solutions. In physics, the energy of a material sometimes jumps to a higher state even though the transition is controlled by a current temperature. The higher the energy that a material has, the more the material jumps to a higher temperature. The step of the cooling schedule represents this process by successively lowering the temperature. At the initial stage, most worse solutions are accepted because the temperature is high. In this stage, the procedure of the algorithm is similar to that of a random search method. As the temperature cools, fewer worse solutions are allowed. At the coldest temperature, very few disruptions are permitted. In practice, a simple parameter is multiplied to a current solution in order to reduce temperature:

$$T = rT^{new} \quad (4.5)$$

where r is a cooling parameter whose value is between zero and one.

The simulated annealing algorithm has been applied to wide areas of research: a vehicle routing (Robuste et al., 1990); a network design problem (Friesz et al., 1992; Meng and Yang, 2002; Drezner and Wesolowsky, 2003); an assignment of aircrew to planned rotations (Lucic and Teodorovic, 1999); and determining berth times and positions of containerships (Kim and Moon, 2003).

The genetic algorithm is an analogous approach to the processes of natural selection and survival of the fittest in the evolution of species. Figure 4-1 shows a conceptual diagram for genetic representation in the algorithm. In this diagram, decision

variables, which are denoted as substrings, consist of a series of genes that are normally encoded in binary digits as zero or one. The collection of the decision variables forms a string, which is a solution at this generation. A string is updated at every generation, imitating an evolutionary process in biology. Three typical operators are applied in this process, namely reproduction, cross-over and mutation.

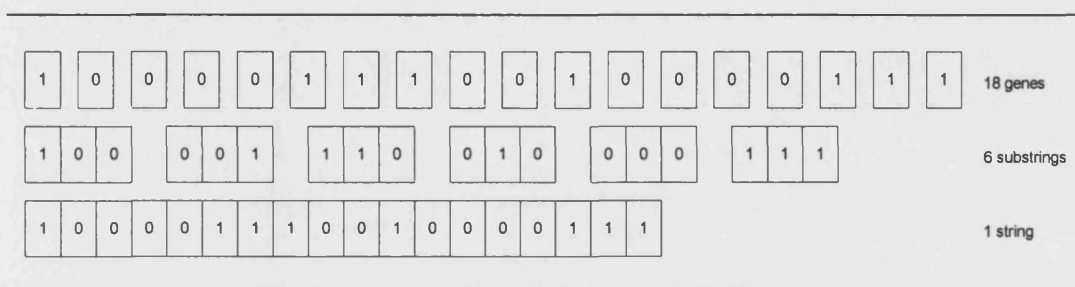


Figure 4-1 A genetic representation, source: Liu and Mahmassani (2000).

Step 0. Initialise a solution $\{\mathbf{x}^0, \mathbf{y}^0\}$; set $k=0$.

Step 1. Evolutionary procedure.

Step 1.1. Reproduction.

Step 1.2. Cross-over.

Step 1.3. Mutation.

Step 2. If a solution satisfies the stopping criteria, then terminate the procedure; otherwise set $k=k+1$ and go to 'Step 1'.

An individual decision variable is evaluated in terms of its fitness to the objective function. The variables that satisfy the predetermined fitness criteria are copied to the next generation; this process is called reproduction. The reproduction step signifies that fitter variables have a higher probability of contributing to the generation of offspring. This procedure is analogous to the law of dominance in biology. The recessive variables are randomly mated to improve the fitness to the objective function; cross-over denotes this process. Finally, the step of the mutation generates random perturbations in the decision variables. This process may enable a solution to jump out from local optima towards the whole search space.

The genetic algorithm has recently been applied to several research areas: finding a global maximum likelihood estimate (Liu and Mahmassani, 2000); a study for parking

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guidance and information systems (Thompson et al., 2001); a network design problem (Lo et al., 2001; Bielli et al., 2002; Drezner and Wesolowsky, 2003; Abu-Lebdeh and Benekohal, 2003; Ceylan and Bell, 2004); optimising highway alignment (Jong and Schonfeld, 2003); a multi-item inventory problem (Chan et al., 2003); and a berth allocation considering service priority (Imai et al., 2003).

4.2.5. Discussion

Existing solution algorithms for bi-level models adopt heuristics. The algorithms solve the problems by alternating the upper and lower levels. In this section, the algorithms have been classified into the deterministic and stochastic types. The probabilistic algorithms suggest different answers every time the algorithms run, whilst the deterministic algorithms offer the same answer when the algorithms calculate solutions. The deterministic algorithms have been subdivided into the iterative and responsive algorithms in terms of their game theoretical context. While the iterative algorithms represent the mutual consistency between the levels, the responsive algorithms consider the first-mover advantage and the reaction of the followers. The characteristics of each type of algorithm are summarised in Table 4-1.

Table 4-1 The characteristics of the existing algorithms

	The deterministic algorithm		The stochastic algorithm
	Iterative types	Responsive types	
Equilibrium	Nash	Stackelberg	N/A
Computation time	Reasonable	Reasonable	Very demanding
Convergence	Not always	Not always	Not always
Solution	Local optima	Local optima	Potentially global

The deterministic algorithm possesses a number of attractive characteristics. Two principal components can be selected. First, the algorithm provides a reasonable framework to analyse the behaviour of decision-makers. The algorithm can represent not only the simultaneous competition but also the leader-follower structure. Secondly, the algorithm is relatively simple in form and convenient to represent. A successive alternating procedure is easily implemented, not demanding too much computational requirement. However, it was proved that the algorithm might not converge (Marcotte, 1981). Even though the algorithm converges, the solution found may not be unique. This is because bi-level programmes are inherently non-convex. Empirical studies, however, have reported that a local solution is likely to be a global answer when the objective function of the upper level is strongly convex with respect to both the decision variables of the upper and lower levels (Yang and Bell, 1998).

The stochastic algorithm would be attractive since the technique might offer a superior solution to the deterministic counterpart. This group, however, has similar

problems to that of the deterministic type owing to the non-convex nature of bi-level programmes. The probabilistic algorithm requires even more computation time than the deterministic algorithm. This is because the stochastic type searches wide areas in the solution dominion than the deterministic algorithm. Furthermore, there is no guarantee of convergence of the algorithm. Moreover, it is difficult to interpret the behavioural context of decision-makers in this category.

To sum up, there is no efficient solution algorithm that guarantees producing a global optimum. The challenge to develop a global search algorithm for bi-level programmes is still open. In fact, the quest is beyond the scope of this study. In the next section, the solution algorithm for the bid-rent network equilibrium models will be considered within the framework of the existing solution-finding approaches.

4.3. The solution algorithm

In the previous section, it was concluded that no existing types of algorithms suggested a global solution with computational efficiency. The difficulty was understood in the context of the non-convex nature of bi-level mathematical programmes. As noted at the end of the previous section, developing a new algorithm is beyond the scope of this study. In this section, a reasonable solution algorithm is proposed for the bid-rent network equilibrium model. This study decided to adopt the iterative algorithm as a solution-finding technique for the model. There are several reasons for the decision. First of all, the iterative algorithm can reasonably represent the behaviour of decision-makers in the relationship between transport and the location of activities. The behavioural context in terms of the best response of households from the framework of the Cournot game is consistent with the procedure of the iterative algorithm. Secondly, the simple form of the algorithm is attractive to implement. The procedure for the successive alternations between the levels is easily visualised. Finally, the iterative algorithm produces an acceptable solution in a reasonable computation time. In particular, the mutually consistent solution from the algorithm is compatible with the definition of the Nash equilibrium that the proposed model intends to find. The solution algorithm suggested is summarised as follows:

Step 0. Initialise a feasible solution $\{q_m^{rs,0}, \varphi_m^{r,0}\}$ and $\{f_{pm}^{rs,0}\}$; set $k=0$.

Step 1. Find the lower level solution $\{f_{pm}^{rs,k}\}$ with $\{q_m^{rs,k}, \varphi_m^{r,k}\}$.

Step 2. Solve the upper level problem $\{q_m^{rs,k+1}, \varphi_m^{r,k+1}\}$ with $\{f_{pm}^{rs,k}\}$.

Step 3. If the stopping criterion is met, then terminate the procedure; if not, set $k=k+1$ and go to 'Step 1'.

The algorithm requires two sets of components for the initial solution. One set of the solution comprises the matrix of the initial flow of spatial interactions between locations and the incipient locational attractiveness. The initial transport impedance between zones represents the other component of the solution. Since the procedure of the solution algorithm proposed in this study is fundamentally heuristic, a good model outcome would be dependent on a good initial solution. This is because the heuristics would be trapped at the dominion of local optima. This would suggest that the initial

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solution would determine the performance of the solution algorithm for the bid-rent network equilibrium model. Hence, a choice of the initial solution would be important to obtain a good result of the model run. Further discussions on this issue and a practical selection for the initial solution are considered in Chapter Five, Case Studies.

At 'Step 1', the lower level represents the minimisation of the net interaction impedance, which has explicitly considered the two indicators for the level of service in the network meaning locational attractiveness and transport impedance. The lower level has described the decision of non-locators in the process of the relationship between transport and the location of activities. The resulting formulation, which has used a variational inequality, has represented the Nash equilibrium.

There are a number of algorithms in the literature for the numerical solution to 'Step 1'. Most algorithms adopt an iterative approach where the algorithms start from an initial feasible solution and then modify and improve the solution. One possible way to classify the techniques is by the level of aggregation for storing the solution at each iteration, namely link-based and path-based algorithms (Bar-Gera, 2002). The link-based or the aggregate algorithm stores the total link flows of all origin-destination pairs. This group is known to require relatively modest memory, but the convergence rate is slow because of so-called the zigzag problem. First proposed by LeBlanc and others (1975) using the Frank-Wolfe (1956) procedure, which had been originally suggested for solving non-linear optimisation programmes, there have been many variants (Evans, 1976; Fukushima, 1984; LeBlanc et al., 1985; Lupi, 1986). The simplicial decomposition routine (Lawphongpanich and Hearn, 1984; Hearn et al., 1987; Larsson and Patriksson, 1992; Lee, 1995) may be understood to be its most general version. The path-based or the disaggregate algorithm stores the flow for each OD pair in its own set of routes. This group requires a large computer memory but is known to achieve better solutions. The main feature of this algorithm is the flow shift method from high cost routes to low cost routes until convergence is obtained (Dafermos and Sparrow, 1969; Schittgenhelm, 1990; Jayakrishnan et al., 1994).

In principle, either type of algorithm can be used to represent 'Step 1'. However, the path-based algorithm is adopted here for several reasons. First, the path-based algorithm more realistically represents the behavioural context of the transport

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decision of households than the link-based counterpart. A household is expected to compare possible net interaction costs. The costs are determined by the difference between the route cost and the locational attractiveness. Secondly, the path-based algorithm is a general approach. The link-based algorithm is equivalent to the path-based algorithm if and only if transport costs are assumed to be additive and link travel times consist of route costs only (Bliemer, 2001; Bliemer and Bovy, 2003). In contrast, the path-based algorithm can flexibly incorporate various cost components besides the link travel time. Finally, the path-based algorithm is easy to implement and achieves a better solution (Nagurney, 1984; Jayakrishnan et al., 1994). Furthermore, the algorithm provides direct convergence information during the execution of the algorithm; the cost and flow of paths at each iteration can be monitored as to whether they satisfy the convergence measure (Schittenhelm, 1990).

The path-based algorithm requires a route enumeration *in priori*. The advantage of the path enumeration is that no additional computation for finding paths is necessary during the execution of the algorithm. This is because alternative routes are computed in advance and stored together with the network. On the other hand, the paths listed could be too large to handle in terms of memory requirements and computation time (Bliemer, 2001). This problem, however, has become less problematic owing to a drastic improvement of computerising devices.

In fact, enumerating all acyclic routes would guarantee to identify all possible alternatives of decision-makers. The all path generation, however, is impractical for a large network because the number of routes grows rapidly with the size of a network. Furthermore, empirical surveys have reported that network users are likely to use only a few routes (Bliemer, 2001). In this context, a subset of all possible paths can be selected as a sufficient choice set for travellers. In this case, rules should be used to decide which routes are included and excluded. Several methods can be considered. The k shortest path algorithm (Shier, 1974) produces routes that require smaller transport costs. This technique, however, is likely to drop significant alternative paths because the number of k is arbitrarily determined. The multi-criteria approach (Ben-Akiva et al., 1984; Battista et al., 1995; Dial, 1996) incorporates a set of factors when the approach selects feasible routes. This approach requires a considerable field survey to identify network specific parameters as well as the behaviour of network

users. In contrast, the network restriction approach (Dial, 1971) is attractive. The Dial algorithm heuristically eliminates unreasonable paths. In this technique, a reasonable path is defined as a set of links that lead away from the origin and toward the destination. Two principal advantages of the restriction approach would be noted as follows: first, this technique requires no additional information to specify reasonable paths; secondly, the heuristics are consistent with the behaviour of network users in the sense that a reasonable user is unlikely to take the more expensive link. Some studies have incorporated a penalty factor into the Dial algorithm to reduce the use of overlapping paths (Cascetta et al., 1996; Russo and Vitetta, 2003). This approach, however, requires an additional procedure to estimate the penalty factor.

Since the solution algorithm for this study has incorporated the route-based routine, a rule for the path enumeration should be decided. This study assumes that all reasonable paths form the alternative routes of network users. This is represented by means of combining the Dial algorithm (1971) and an all path enumeration technique. The process has two stages: first, the Dial algorithm reduces a network eliminating unreasonable links; then, the technique of an all path generation searches all feasible paths in the restricted network by a topological order. The procedure is summarised as follows: let $o(r)$ be the travel time from an origin node to a node r along a minimum travel time path and $d(s)$ be the travel time from a node s to a destination node along a minimum path; then, the two-stage procedure is given by

Step 1. Compute a minimum travel time from an origin node to all other nodes.

Step 2. Compute a minimum travel time from each node to a destination node.

Step 3. For each link (r, s) compute the link likelihood $L(r, s)$, where

$$L(r, s) = \begin{cases} e^{\theta[o(r)-o(s)-t(r,s)]} & \text{if } o(r) < o(s) \text{ and } d(r) > d(s) \\ 0 & \text{otherwise} \end{cases} \quad (4.6)$$

Step 4. Reduce a network eliminating links whose link likelihood is zero.

Step 5. Enumerate all possible paths using the reduced network.

Even though the number of alternative paths is considerably reduced in the restricted network, the enumerated routes would be too many to handle. In particular, handling

all enumerated alternatives would be intractable in a large-sized network with which land-use transport studies normally deal. This is because the number of reasonable paths grows exponentially with the size of the network. Furthermore, the number of paths used at equilibrium is quite small relative to the total number of alternatives. To tackle this problem, the column generation algorithm (Leventhal et al., 1973) is incorporated. The algorithm does not carry all enumerated paths during the sequence of calculations. The technique generates and drops routes whenever necessary. The procedure of the column generation algorithm can be described as follows:

Step 0. Start with any subset of paths $P_{sub}^{rs,0}$; set $k=0$.

Step 1. Generate a column if a minimum cost path found outside the subset of used paths at iteration k ; $P_{sub}^{rs,k} = P_{sub}^{rs,k-1} \cup \{p_{min}^{rs,k}\}$.

Step 2. Drop a column if a path carries no flows at iteration k ; $P_{sub}^{rs,k} = P_{sub}^{rs,k-1} - \{p_0^{rs,k}\}$

Step 3. If convergence is obtained, then stop; otherwise go to 'Step 1'.

In summary, this section has discussed some components that compose the procedure of the solution algorithm for the lower level of the bid-rent network equilibrium model. The Dial algorithm (1971) is used to reduce a network. The all path generation technique identifies the alternative routes of network users within the restricted network. The path-based algorithm is used to find a network equilibrium. In this process, the column generation technique (Leventhal et al., 1973) helps to execute an efficient calculation. This procedure can be summarised as follows:

Step 1. [The lower level] Find the lower level solution $\{f_p^{rs,k}\}$ with $\{q^{rs,k}, \phi^{r,k}\}$.

Step 1.0. Enumerate all reasonable paths between an OD pair $\{r,s\}$.

Step 1.1. For each OD pair $\{r,s\}$, perform an all-or-nothing assignment which yields initial path flows $\{f_p^{rs,0}\}$ and costs $\{\tilde{c}_p^{rs,0}\}$.

Step 1.2. For each OD pair, repeat the following steps:

Step 1.2.1. Update the memory route set, i.e. drop paths that carry no flows p_0^{rs} ; generate columns if a minimum cost path p_{min}^{rs} is found outside the memory route set.

Step 1.2.2. Find a minimum cost path p_{\min}^{rs} and a maximum cost path p_{\max}^{rs} in the memory route set.

Step 1.2.3. Transfer flows from a maximum cost path to a minimum cost path in order to equilibrate between p_{\min}^{rs} and p_{\max}^{rs} .

Step 1.3. Convergence Test. If convergence is obtained, then terminate the procedure; if not, go to 'Step 1.2'.

The proposed algorithm is not feasible to solve the multiclass bid-rent network equilibrium model. As discussed in Chapter Three, the multiclass model considers the asymmetric interactions between user classes. Solution algorithms in the literature for this issue usually replace the original asymmetric structure with the augmented symmetric problem iteratively. Two typical approaches can be considered, namely the projection method and the relaxation (diagonalisation) technique. In general, the projection approach solves a sequence of linear symmetric problems at iteration (Dafermos, 1980; Nagurney, 2000; Nagurney and Dong, 2002a; Nagurney and Dong, 2002b). The relaxation algorithm solves a sequence of non-linear symmetric problems (Dafermos, 1982b; Mahmassani and Mouskos, 1988). Intuitively, the relaxation algorithm is likely to produce a better solution than the projection routine. This is because the relaxation algorithm calculates the exact solution at iteration whilst the projection technique calculates an approximate solution. Comparative studies have confirmed this expectation: the relaxation algorithm outperformed the projection method in terms of convergence (Fisk and Nguyen, 1982); both algorithms produced the same output using a linear travel cost function, but the relaxation algorithm showed a significant better performance when highly non-linear cost functions were used (Nagurney, 1984). For this reason, the relaxation technique was used as a solution-finding method for the multiclass bid-rent network equilibrium model.

The relaxation algorithm requires one more outer loop that describes a class-specific behaviour. This is addressed by solving the following variational inequality:

$$\sum_m \sum_{rs} \sum_p \tilde{c}_{pm}^{rs} (f_{pm}^{rs*}, \bar{\varphi}^r | \bar{\mathbf{f}}_{-m}) (f_{pm}^{rs} - f_{pm}^{rs*}) \geq 0 \quad (4.7)$$

where $\bar{\mathbf{f}}_{-m}$ is the vector of the fixed path flow of household classes except a class m .

Convergence of the relaxation algorithm was proved under the condition that the Jacobian of the link-travel-time function is positive definite (Dafermos, 1982b). If the algorithm converges, the solution found would be the equilibrium flow pattern (Sheffi, 1985, p. 217). Convergence of the relaxation algorithm may be achieved even though the sufficient condition of Dafermos is violated (Heydecker, 1983; Mahmassani and Mouskos, 1988). However, the strong assumptions for the condition of convergence would suggest that the problems would have multiple local optima.

At ‘Step 2’, the model describes the locational decision of households in the competition for residence. The hedonic-based random bid-rent function has been suggested for representing the behaviour of locators. The function has been discussed to successfully suggest the relationship between the decision-making process of households and the unique characteristics of locations. The resulting formulation has been understood as an equivalent to the oligopolistic Cournot game for locations.

The parameters in the bid-rent function can be estimated by the maximum likelihood procedure. A conventional assumption is that the system component of the utility function $V_{H \in m}^{r\sigma}(\cdot)$ is linear in unknown parameters β , namely $V_{H \in m}^{r\sigma}(\cdot) = \beta^T V_{H \in m}^{r\sigma}(\cdot)$.

The log likelihood function is given by as follows:

$$LL = \sum_r \sum_m \delta_m^r \left[\beta^T V_{H \in m}^{r\sigma}(\cdot) - \ln \sum_{m'} \exp \{ \beta^T V_{H \in m'}^{r\sigma}(\cdot) \} \right] \quad (4.8)$$

where r is a distinct observation unit of a residential location and

δ_m^r is an indicator variable; the value is either one or zero.

Provided the data $\{\varphi_m^r, u_{H \in m}^{r\sigma}\}$, $\forall m \subset M$ are not multicollinear, they will satisfy a full-rank condition. The condition guarantees that the Hessian $\partial^2 LL / \partial \beta \partial \beta'$ is negative definite. Then, the log likelihood function is strictly concave and the parameter vector that satisfies $\partial LL / \partial \beta = 0$ is a unique solution (McFadden, 1973).

Many iterative methods exist that begin with a starting value $\{\beta^0\}$ and find a point that is close to the maximum. Here, the standard Newton-Raphson method is adopted.

The technique is a gradient method that uses a linear approximation of the Taylor series. The procedure of ‘Step 2’ is summarised as follows:

Step 2. [The upper level] Solve the upper level problem $\{q_m^{rs,k+1}, \varphi_m^{r,k+1}\}$ with $\{f_{pm}^{rs,k}\}$.

Step 2.0. Estimate the parameter $\{\beta\}$ with the maximum likelihood procedure.

Step 2.0.1. Choose an initial value of the parameters $\{\beta^0\}$.

Step 2.0.2. Solve the following linear function

$$\beta^{k+1} = \beta^k - [\nabla^2 LL(\beta^k)]^{-1} \nabla LL(\beta^k) \quad (4.9)$$

Step 2.0.3. If convergence is attained, then terminate the step; if not, go to ‘Step 2.0.1’.

Step 2.1. Update the demand matrix.

Step 2.1.1. For each OD pair $\{r, s\}$, update the class-specific spatial interaction flow of household members of locators using

$$\bar{q}_m^{rs,k+1} = \bar{D}_m^s \times Pr^{rs,k+1}(m | z) \quad (4.10)$$

where $\bar{D}_m^s = \sum_h \sum_r h_m^{rs}$, $\forall m, s$ and $P^{rs}(\cdot)$ is the probability of the spatial interaction between an OD pair $\{r, s\}$.

Step 2.1.2. For each location r, m , update the locational attractiveness

$$\text{using a logsum formula, } \varphi_m^{r,k+1} = \ln \sum_{m'} \exp[\beta^{T,k+1} V_{H \in m'}^{r\sigma}(\cdot)].$$

Step 2.1.3. Set the demand matrix $\{q_m^{rs,k+1}, \varphi_m^{r,k+1}\}$ where $q_m^{rs} = \bar{q}_m^{rs} + \tilde{q}_m^{rs}$.

At ‘Step 3’, the stopping criterion is checked. Some rules for the termination can be considered. A preferable criterion would be a measure that can represent behavioural or economic meanings of the bid-rent network equilibrium model rather than mere mathematical constructs. In this study, the relative difference between two consecutive values of the maximum bid-rent from iterations is adopted as the criterion. This is because the model focuses on the spatial changes by the bid-auction process of households. The bidding is expected to change the maximum rent for each location. The maximum rent is interpreted as a measure for locational attraction. The

changes in the attractiveness invoke the behavioural variance of households, which eventually changes the urban structure in terms of spatial demands and zonal density.

$$\frac{\sum_m \sum_r |\varphi_m^{r,k+1} - \varphi_m^{r,k}|}{\sum_m \sum_r \varphi_m^{r,k}} \leq \kappa \quad (4.11)$$

where κ is a predetermined constant for the termination of the algorithm.

In summary, this section has suggested the solution algorithm for the bid-rent network equilibrium model. The path-based routine finds the equilibrium solution of the lower level. In the upper level, the Newton-Raphson procedure is used to estimate the parameters of the hedonic-based stochastic bid-rent function. The two procedures are incorporated in the heuristic process, which solves the proposed bi-level model. This algorithm is expected to produce a mutually consistent solution that is compatible with the Nash equilibrium which the developed model wants to find.

4.4. Numerical examples

The following simple numerical examples illustrate the operation of the bid-rent network equilibrium model. This is considered in terms of a selfish game of households for residing at locations. The competition represents the nature of the relationship between transport and the location of activities. Three principal data are required for the simulation. The data are the representation of transport network, the matrix of locational values, and the spatial interactions of trip-makers.

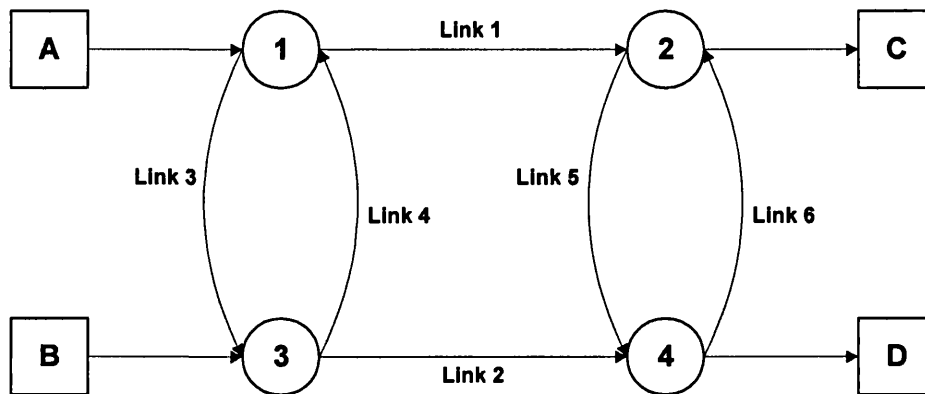


Figure 4-2 A network for the numerical examples (Bell and Iida, 1997)

Table 4-2 Link capacity (unit: veh/min)

Link	Capacity	Link	Capacity
1	30.0	4	20.0
2	30.0	5	20.0
3	20.0	6	20.0

Table 4-3 Path specification

Path	A set of links	Path	A set of links
$p(1)$	(1)	$p(5)$	(2,6)
$p(2)$	(3,2,6)	$p(6)$	(4,1)
$p(3)$	(1,5)	$p(7)$	(2)
$p(4)$	(3,2)	$p(8)$	(4,1,5)

Figure 4-2 shows the example network (Bell and Iida, 1997, p. 153). The network consists of four nodes and six links. Specific information of link capacity is summarised in Table 4-2. The network has eight distinct acyclic paths whose references are shown in Table 4-3, and four origin-destination pairs. The link cost functions, which are shown in Table 4-4, are defined as having the form of US BPR (1964) type. The base model has a single value of travel time across household classes, i.e. $w = 1.12$. The multiclass model I associates with class specific parameters

to the cost functions, namely $w^m = 1.12$ and $w^c = 2.03$ (Lam and Huang, 1992b). The cost functions for the bi-dimensional multiclass model are specified in terms of decision-makers' modes of travel (Kim, 1997), where t and c denote transits and cars respectively. This model also associates with the same value of travel time as the multiclass model I, namely $w' = 1.12$ and $w^c = 2.03$. These weighting factors are required for the compatible representation between the upper and the lower levels, as discussed in Chapter Three. This is because the two decision variables, which are transport impedance and locational attractiveness, are specified in a different unit.

Table 4-4 Link cost functions

Model	Cost function
The bid-rent network equilibrium model	$t_a = 80 \left(1 + 0.15 \left(\frac{v_a}{C_a} \right)^4 \right)$
The single dimensional multiclass bid-rent network equilibrium model (Multiclass I)*	$t_a^m = w^m \cdot 80 \left(1 + 0.15 \left(\frac{v_a}{C_a} \right)^4 \right)$
The bi-dimensional multiclass bid-rent network equilibrium model (Multiclass II)**	$t_a^c = 80 \left(1 + 0.15 \left(\frac{v_a^c + 1.5v_a'}{C_a} \right)^4 \right)$ $t_a' = 75 \left(1 + 0.29 \left(\frac{v_a^c + 1.5v_a'}{C_a} \right)^{3.33} \right)$

*Lam and Huang (1992b)

**Kim (1997)

The initial locational attractiveness is assumed to be $[20.0, 40.0]^T$. The value is represented as unit prices. The initial spatial distribution, which is shown in Table 4-5, consists of two classes of households, namely m_1 and m_2 . Both groups are assumed to be almost evenly distributed and 20% of them are supposed to consider moving residence. m_1 is defined as a lower income group in the base and single dimensional models. On the other hand, m_2 is defined as a group of households that are higher-income travellers. In these models, the level of income is assumed to be a sufficient factor that can distinctly classify decision-makers and all trip-makers are supposed to use cars for travels. In the multiclass II model, m_1 is identified as a lower-income and transit-using group, and m_2 is specified as a higher-income and car-using group.

Table 4-5 OD demand

OD pair	Demand		OD pair	Demand	
	m_1	m_2		m_1	m_2
A-C	25	25	B-C	24	26
A-D	26	24	B-D	25	25

The original data in the Bell and Iida example are designed for analyses of travel demands. Thus, trips are considered to be demands for the locations of activities given the residences that are referred to as origins. It was decided to convert the data of travel demands into those of spatial interactions appropriate to the purpose of this study. Thus, this study interprets the OD data conversely to the existing setting. In other words, the OD data are interpreted as households' choice of residential locations given the primary activity locations, which are virtually regarded as origins. The Bell and Iida example has the aggregate OD data that is required at the lower level, but there are no disaggregate data representing locators. The locating data are used at the upper level to represent the changes in urban structure, which is the primary purpose that the proposed model tries to address. In this numerical example, the data for locating households are generated by the inverse transform method (Law and Kelton, 1991, pp. 462-521). This method requires a predetermined distribution function that is appropriate for generating variables. In this simulation, the Gumbel distribution is considered for the hedonic-based random bid-rent model. For description detailed for the inverse transform method, see Appendix II.

The procedure for the solution algorithm was coded in Visual Basic 6.0 on a Dell Optiplex GX240 personal computer using Intel® Pentium® IV CPU 1.80GHz. The algorithm adopts the stopping criterion as the changes in the absolute value of the maximum bid-rents across locations at two successive iterations. Specifically, the algorithm stops when the relative difference between two successive maximum bid-rents is less than 10^{-5} . In the lower level, the calculation terminates when the gap value in terms of mean excess costs per a trip-maker is less than 10^{-5} . In the upper level loop, the convergence measure is that the proportional changes in the functional value of log-likelihood is less than 10^{-5} .

Tables 4-6, 4-7, and 4-8 show the results of model runs. The term 'before' means the initial settings in terms of the spatial distribution of households and the level of

service in the network. The term ‘after’ represents the changes in the urban structure with respect to the household allocations and the network performance indicators. These outputs are understood in this study to result from the selfish locational competition of decision-makers. However, it should not be interpreted as an equilibrium urban structure. To address the equilibrium, an economic relocation model is required; this issue is beyond the scope of the study.

Figure 4-3 shows the convergence of the algorithm. The base model required eight, the multiclass model I eleven, and the second multiclass model ten iterations respectively for obtaining the convergence. The bid-rent network equilibrium model showed a slight oscillation while all the models smoothly converged.

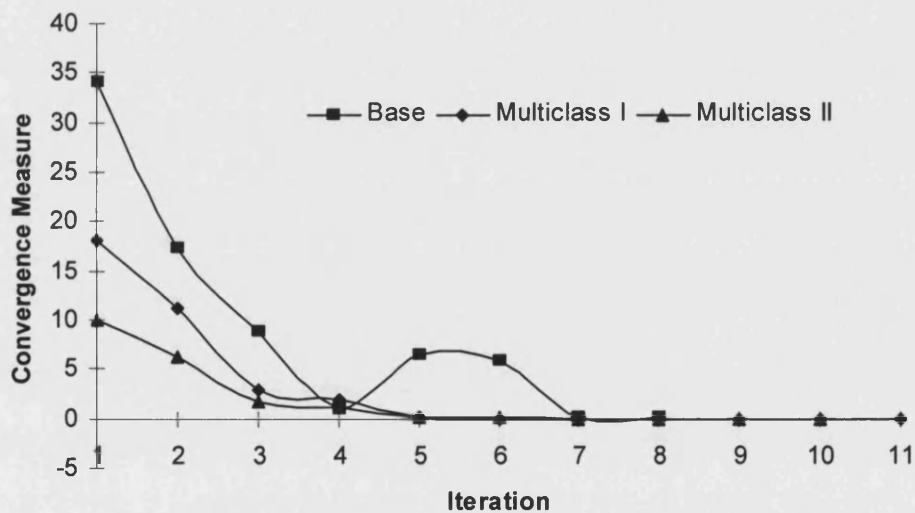


Figure 4-3 Convergence of the algorithm

Table 4-6 shows the results of the bid-rent network equilibrium model and the resulting network performance indicators of transport impedance and locational attractiveness. The changes in the indices show that location C is more attractive than location D. Location C is cheaper in the cost and higher in the attraction than location D. The changes in the indices have caused that households have bid more rent for location C than location D. This has created more people have converged at location C. One of the interesting results is that the lower income group is more sensitive than the higher income group in locating residence. A relatively large number of m_1

changes residence, but m_2 do not. This is because the perceived locational attractiveness and interaction costs of m_2 are lower than those of m_1 .

The results of the multiclass bid-rent network equilibrium model I are summarised in Table 4-7, and the second multiclass model is in Table 4-8. In both cases, m_1 moved to the location D and m_2 converged to the location C. These outcomes could be explained as follows: even though there have been some changes in the net interaction impedance, the relative difference of the value between locations is not very great; on the other hand, the locational attractiveness has been considerably changed; these two asymmetric trends has driven the lower income group m_1 converged in cheaper location D, and the higher income group m_2 converged in expensive location C.

Table 4-6 Results of the bid-rent network equilibrium model

OD pair	Spatial distribution				Impedance				Attractiveness	
	before		after		before		after		before	after
	m_1	m_2	m_1	m_2						
A-C	25	25	30.0	26.2	166.9		144.2		20.0	26.6
A-D	26	24	21.0	22.8	177.1		157.1		40.0	12.5
B-C	24	26	26.3	26.4	177.1		142.4		20.0	26.8
B-D	25	25	22.7	24.6	166.9		158.9		40.0	12.5

Table 4-7 Results of the multiclass bid-rent network equilibrium model I

OD pair	Spatial distribution				Impedance				Attractiveness	
	before		after		before		after		before	after
	m_1	m_2	m_1	m_2	m_1	m_2	m_1	m_2		
A-C	25	25	22.9	27.5	166.9	302.5	164.2	316.3	20.0	19.5
A-D	26	24	28.1	21.5	177.1	320.9	180.5	341.7	40.0	15.7
B-C	24	26	24.0	26.5	177.1	320.9	176.3	337.5	20.0	19.5
B-D	25	25	25.0	24.5	166.9	302.5	168.9	320.5	40.0	15.7

Table 4-8 Results of the multiclass bid-rent network equilibrium model II

OD pair	Spatial distribution				Impedance				Attractiveness	
	before		after		before		after		before	after
	m_1	m_2	m_1	m_2	m_1	m_2	m_1	m_2		
A-C	25	25	23.4	27.5	163.3	306.5	164.6	337.7	20.0	19.2
A-D	26	24	27.6	21.5	166.1	309.1	171.7	344.5	40.0	15.8
B-C	24	26	23.5	26.6	166.1	309.1	167.7	340.5	20.0	19.2
B-D	25	25	25.5	24.4	163.3	306.5	168.5	341.7	40.0	15.8

4.5. Conclusion

This chapter has proposed the iterative algorithm for the solution-finding technique of the bid-rent network equilibrium model. The heuristic algorithm combined the path-based routine for calculating the equilibrium solution to the lower level with the Newton-Raphson procedure for estimating the parameters of the hedonic-based random bid-rent function in the upper level. The solution algorithm was successfully implemented in the numerical examples, which were designed to illustrate the operation of the bid-rent network equilibrium model.

The solution algorithm has some attractive advantages. First of all, the algorithm reasonably represents the behaviour of decision-makers in the game for occupying locations. The successive alternations between the levels, exchanging a solution of each level, are analogous to the best responsive competition of households for locations. Furthermore, the resulting mutually consistent solution of the algorithm is compatible with the definition of the Nash equilibrium, which has been suggested for representing the behaviour of households. Secondly, the solution algorithm is simple in form and relatively easy to implement. These characteristics enable the solution algorithm to be visualised. Furthermore, the flexible structure of the algorithm allows incorporation of appropriate subroutines for the levels. Finally, the algorithm requires reasonable computation resources. In other words, the heuristic method produces an acceptable solution in a reasonable calculation time. The solution found, however, would be local optima because of the non-convexity of the bi-level structure.

The quest to find the global optimum in bi-level programmes is a continuous challenge. The inherent non-convexity of the programmes requires considerable computation time. The solution, which could be one of the local optima, is the even worse disadvantage of bi-level programmes. As discussed in the section on the review of existing solution algorithms, no existing algorithm suggests the global optimum in a reasonable computation time. Developing an efficient solution algorithm that guarantees the global optimum is still an open challenge in transport studies.

5. Case Studies

5.1. Introduction

This chapter deals with an application of the bid-rent network equilibrium model to a real network. The chapter describes the process of simulations for the application and discusses the results of the model runs from the application. The objective of the case study is to demonstrate the ability of the bid-rent network equilibrium model to represent the relationship between transport and the location of activities, and to explore policy implications of the model involving various transport issues.

The city of Ansan in Korea was chosen as a study area. There were several reasons for the selection. More than anything else, the size of the city was attractive. Ansan is a medium-sized city that is not very difficult to represent but would show interesting aspects with respect to interactions between land-use and transport. Another important reason was data availability. One of the major considerations in the use of models was operational. This meant that applications of models should be designed to consider the data available. The city government of Ansan conducted a transport master plan with a comprehensive survey in 1998. Hence, the base year of the simulation was set at the same year as used in the master plan. The simulation in this study tried to maximise the use of the data surveyed and to test the bid-rent network equilibrium model as far as the data were available. Nevertheless, the travel mode based multiclass bid-rent network equilibrium model could not be tested because of the lack of essential data. Thus, the case study could not represent changes in the behaviour of households and variations in urban structure generated by a multimodal transport system.

In the next section, a brief introduction to the city of Ansan is presented. The introduction is followed by a description of the data that were used in the application. In the subsequent section, a design for the simulation is considered. This study adopted the same strategy for testing the bid-rent network equilibrium model as used in the study of ISGLUTI (Webster et al., 1988) and the research of MASTER (Mackett, 1992; 1993a). Hence, the model was applied in the base and the policy runs. The results of the simulation of the model are presented in the next three sections. The outcomes of each run are obtained in terms of the spatial distribution of households,

which are discussed against the class-specific behaviour of the decision-makers. The behaviour is demonstrated using the network performance indicators of transportation costs and locational benefits. Then, comparisons between the results of the runs are considered. Finally, some conclusions are drawn.

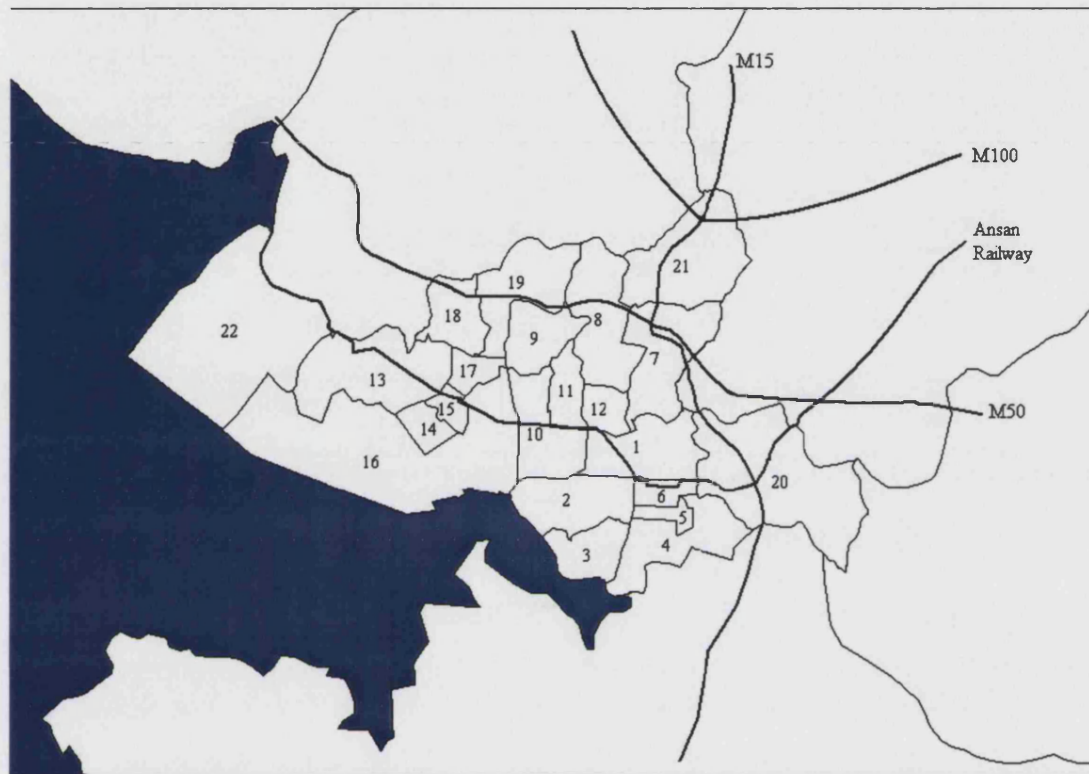


Figure 5-1 The study area-the city of Ansan

5.2. The study area

The city of Ansan is located in the mid west of the Korean peninsular. The southwestern side of the city is bounded by the Yellow Sea. The eastern side of the city borders upon the Seoul metropolitan area. Ansan is about 20 miles away from Seoul. Motorway 15, which is the main west coastal corridor of Korea, connects Ansan and Seoul. Motorway 100, which is the Greater Seoul Outer Ring Road, passes through north Ansan. An electric railway is running between the two cities. The north of Ansan is on the border of the Incheon metropolitan area, which is Korea's fourth largest city. The two cities are around 23.6 miles apart. Motorway 50 connects Ansan to the cities of Singal, Siheung, and Incheon; for a geographical layout of the transport network, see Figure 5-1. These convenient transport networks of the North-South corridor and its East-West counterpart have made Ansan a regional industrial centre.

Ansan was the first planned city in Korea during the 1970s. The city was designed to be self-sufficient in terms of economic and service activities. In accordance with the Banwol Construction Plan, which was Korea's industrial complex creation plan, and the Population Dispersion Policy from Seoul, the Korean government developed the west coast industrial belt from Ansan to Asan, which made a conurbation along the west coast. Owing to a series of plans together with an efficient transport infrastructure, Ansan is a centre of production and a hub of distributive trades.

Table 5-1 Some indices of Ansan

	1986	1998
Area (km ²)	74.26	144.77
Population (1,000)	127	552
Number of households	31,162	193,736
Car ownership	4,429	135,750

Source: Ansan transport master plan (1998)

Since the establishment of the city of Ansan in 1986, the city has experienced rapid economic growth accompanied with the population increase. As shown in Table 5-1, the area has almost doubled. More than four times of the population growth has been recorded while there has been more than a six-fold increase in the number of households. The difference means that more nuclear families reside in Ansan, which can also be observed in many other cities in Korea. Car ownership shows more than a

30-fold increase. However, the city was stable in terms of population and economic growth in the 1990s. In the early 1990s, the rate of growth slowed down and there have been no significant changes since 1995.

Table 5-2 Land-uses in Ansan (1998)

	km ²	%
Residential areas	20.67	14.28
Retail areas	2.75	1.90
Manufacturing areas	14.30	9.88
Open space	107.06	73.95
Total	144.77	100.00

Source: Ansan transport master plan (1998)

Table 5-2 shows land-uses in Ansan. While open space occupied more than 70 percent of the Ansan area, each land-use shows some geographical trends. Residential areas are mostly spread around the periphery of the city centre. Retail activities exhibit centralisation. The activities accumulate in the oldest area, which is currently a part of the city centre. In the case of manufacturing, two principal factory districts can be found. The areas are on the border of the Yellow Sea.

To sum up, in common with most other cities around the capital city of Seoul, Ansan has experienced rapid growth, but the process has been taking place in an organised fashion. The government policy for Ansan to be self-sufficient has made the city have relatively less interaction with Seoul and Incheon compared with other satellite cities in the Seoul metropolitan area. This has established Ansan as a regional centre with a high proportion of self-employment in the production and distributive trades.

5.3. Data for the application

5.3.1. Introduction

This section presents a set of data used in the simulation for the application of the bid-rent network equilibrium model. The simulation tried to maximise the use of the data surveyed from the Ansan transport master plan (1998). Some additional data were collected. This section begins by describing the spatial system for the case study. The zoning system consists of 22 internal zones and 7 external zones. The model requires disaggregate data for the upper level and aggregate data for the lower level respectively. Transport supplies include network for the physical transport infrastructure and cost functions for representing transport impedance in spatial interactions. Initial network performance indices are given for the application.

5.3.2. The spatial system

A definition of the spatial system is one of the core tasks in the application of this kind of model. Since the model in this study assumes that households compete for locations, it may be ideal to assign a detailed spatial reference to each individual. However, it is unrealistic to set a household too great locational detail. The aim of the application is not so much identifying an exact spatial location for an individual but suggesting a geographic system that allows households to perceive spatial variations between alternative locations. Therefore, it is useful to define the spatial system for households to distinguish the zones. In principle, the spatial units should reflect the areas that are regarded as homogeneous by households. However, this task is very complex and in fact beyond the scope of this study. In practice, it was decided to adopt the same zoning system as was used in the Ansan transport master plan (1998).

The spatial system adopted is shown in Figure 5-1. The system consists of 22 internal zones and 7 external zones; two external zones are not represented in the diagram. It is obvious that several zones are smaller than the others. It can be also seen that the two zones that border on the Yellow Sea are rather larger than desirable. Thus, some modifications to the zoning system for identifying desirable geographic sizes would be necessary. However, the modification requires considerable effort to change the zoning system in the simulation design. This is mainly because the spatial opportunity

would have to be redefined. Most data available are in a format that satisfies the zoning system given. The adjustment does not necessarily led to better outcomes.

It is important to note that there would be some households that live inside the system but their primary activity locations are located outside the system or vice versa. Even though they contribute to the determination of transport impedance and locational attractiveness in the system, the OD table does not capture the volumes. Thus, the simulation may offer underestimated transport costs and locational benefits. In particular, the underestimation causes some significant problems when congestion is considered. However, this unsatisfactory aspect does not matter very much in this study because the unrepresented volumes are mainly related to the external zones.

5.3.3. Data of spatial interactions

In the simulation of the model, two kinds of spatial interaction data are needed. First, the upper level requires disaggregate data to represent the locational behaviour of an individual household. The disaggregate data are used to calculate the probability for the distribution of households across the area and to determine the vector of locational attractiveness. The other one is an aggregate OD matrix that is necessary in the lower level. The matrix is used to evaluate travel time for spatial interactions. The transport cost found in the lower level, together with the locational attraction determined in the upper level, is used to define the net interaction impedance.

It was not feasible to obtain the spatial interaction data of households that were exactly appropriate for the purpose of the model. Specifically, there were no data available about the preference of households on the locational characteristics, the practical residence choice of households, and factors that represent the relationship between transport and the location of activities. Thus, it was decided to convert data of travel demands into those of spatial interactions. For example, data on trips to workplace were interpreted as the residential location choice of households given workplace locations; it is noted that trips in the travel demand data are considered to be demand for activity locations given residences that are referred to as origins.

As for the disaggregate data, data from each household's travel diary were used. The survey was conducted for the Ansan transport master plan (1998). In the survey, a total of 4,000 households were selected by means of random stratified sampling. The number of samples was equivalent to two percent of the total number of households in the city. The effective sample size given in the master plan was 2,825. In this study, 1,536 samples were used after the cleaning process. Most of the samples excluded had missing items that were necessary for the simulation.

Table 5-3 Monthly income by deciles (unit: ₩/Month)

Class	%	Blue Collar	White Collar	Overall
Class 1	~10	562,741	872,355	717,548
	~20	1,005,431	1,342,287	1,173,859
	~30	1,255,177	1,608,671	1,431,924
Class 2	~40	1,481,475	1,857,209	1,669,342
	~50	1,703,445	2,128,427	1,915,936
	~60	1,951,006	2,412,185	2,181,596
	~70	2,252,089	2,732,076	2,492,083
Class 3	~80	2,628,348	3,117,683	2,873,016
	~90	3,193,028	3,694,052	3,443,540
	~100	5,294,871	6,103,192	5,699,032
Average		2,133,115	2,587,904	2,360,510

Source: Korea National Statistical Office (1998)

Table 5-4 Number of samples specific to the user classes

Class	Income (10 ⁴ ₩/Month)	Number of samples
Class 1	~50	36
	50~100	279
	100~150	414
Class 2	150~200	630
	200~250	108
Class 3	250~300	30
	300~350	39
Total		1,536

The disaggregate data were divided into three household classes. The criterion used was the level of income, as discussed in the development of the model. It was decided to adopt the same threshold values as suggested by the Korea National Statistical Office for the classification; the reference tables can be found in Table 5-3 and Table 5-4; Table 5-3 shows the monthly income by deciles in Korea and Table 5-4 represents the equivalent number of samples in Ansan. The number of samples for the third class is rather smaller than that of the others. Other classification rules might be

incorporated to make the number of samples even across classes. However, the incorporation would make the classified groups inhomogeneous.

Table 5-5 Ratios of trips with respect to purpose

Home-based trips (%)			Non-home-based trips (%)	
School	Workplace	Return home	Business	Other
11.97	21.19	46.82	6.53	13.49

Source: Ansan transport master plan (1998)

To make the aggregate OD table, home-based trips were summed. This was because the home-based trips could be interpreted as primary spatial interactions in relation to residential locations. However, the matrix made had one crucial unsatisfactory aspect. Namely, non-home-based trips were not considered in the evaluation of travel time even though the trips obviously contributed to the determination of transport costs. In particular, these demands should be considered when the effect of congestion is involved. It was not feasible to explicitly address the non-home-based trips. As an alternative, it was assumed that the non-home-based trips were loaded on the background network that was not explicitly represented in the transport network for the simulation. These demands may be termed as background traffic.



Figure 5-2 Ansan transport network

5.3.4. Transport supply

Transport supply in the application is represented by a network and link performance functions. The network describes physical transport infrastructure and the functions address travel impedance for spatial interactions. The supply of transport, together with transport demands, determines travel time in the network, which is one component that defines the net interaction impedance for spatial interactions; the other component is locational attractiveness that is determined in the upper level.

Table 5-6 Roads in Ansan (unit: *m*)

	Length (%)	Number of lanes			
		2	4	6	8
Nat'al express	14,389.0 (2.0)				14,389.0
National high	40,194.0 (5.7)			6,578.0	33,616.0
Provincial	16,000.0 (2.3)	16,000.0			
City-county	637,993.6 (90.0)	409,429.8	144,591.7	62,548.8	21,423.3
Total	708,576.6 (100.0)	425,429.8	144,591.7	69,126.8	69,428.3

Source: an internal document, the division of Construction and Transport of Ansan (1998)

Table 5-7 Link specifications

Link	Node		Free flow travel time (min)	Capacity (veh-min)
	From	To		
1	1	2	1.280	25.000
2	1	5	1.840	25.000
⋮	⋮	⋮	⋮	⋮

The network is considered for the car mode. Specifications of roads in Ansan are shown in Table 5-6. It is obvious that city-county roads are more dominant than any other type of road. The city-county roads occupy around 90 percent of roads in the study area. It can be also seen that the national expressway and the national highway are mainly used as trunk roads in the city of Ansan. This can be inferred by the fact that the two types of roads accommodate more than six lanes.

Figure 5-2 shows the Ansan transport network for the case study. This is a modified version of the network that was used in the Ansan transport master plan (1998), improving some components. The proposed network represents the internal zones, which consist of 484 OD pairs. The connection between the internal and external areas is simpler. It was assumed that the connection was represented with one single link that had infinite capacity. Each link in the network has a bundle of characteristics

in terms of link number, incoming node, outgoing node, free flow travel time, and capacity. The typical specification for link information is shown in Table 5-7.

It is worth stressing that not every road is represented in the network. It is almost impossible to represent the entire minor background network even though the background network carries some volumes of travel. This may generate underestimated travel time. As discussed in 5.3.3, this study assumed that the unrepresented network carried the entire background traffic; the background traffic was defined in the previous section as the aggregate sum of non-home based journeys that were not explicitly represented in the OD matrix.

In addition to the physical details, each link has a link performance function. The level-of-service function includes factors that influence travellers in making transport decisions and determines the travel time for spatial interactions. In fact, there are many cost functions to calculate transport impedance, but it was decided to adopt the standard US BPR function (1964). The BPR function has been successfully used in many applications. This is mainly because the function has a simple form but reasonably represents the theory of traffic flow. The dual advantages mean the efficiency of computation and the representation of traffic flow respectively. Since the link performance function should be evaluated numerous times during an equilibration process, a complicated form may impose a significant computational burden. Furthermore, a complicated form does not necessarily produce better outcomes. The US BPR function is given by

$$t_a = t_a^0 \left(1 + 0.15 \left(\frac{v_a}{C_a} \right)^4 \right) \quad (5.1)$$

where t_a is the travel time on a link a ;

t_a^0 is the free flow travel time on a link a ;

v_a is the traffic flow on a link a ; and

C_a is the capacity of a link a .

The multiclass bid-rent network equilibrium model tries to describe class-specific behaviour in making transport decisions. This requires a class-specific cost function.

The function is represented by means of incorporating parameters that identify a class-specific value of travel time (van Vliet et al., 1986).

$$t_a^m = w^m \cdot t_a^0 \left(1 + 0.15 \left(\frac{v_a}{C_a} \right)^4 \right) \quad (5.2)$$

where w^m is a weighting factor that is associated with an equivalent travel time unit for a value of travel time of a household class m .

In calibrating the weighting parameters, it was decided to adopt a simplified method suggested by the Korea Development Institute (1999). The Institute, in the report of an extensive survey, notes that value of travel time can be approximated with an equivalent 130 percent income level per hour, even though many factors should be considered for an exact evaluation of the value. Following this simplified formula, the values were calibrated by dividing 130 percent average monthly income by average monthly working hours. For the single class model, the overall monthly income was used to calibrate the parameter. The detailed references are shown in Table 5-8.

Table 5-8 Value of travel time for user classes (unit: ₩/hr)

	Monthly income ¹ (₩)	Monthly working hours ²	Value of travel time
Class 1	1,009,300	218.5	6004.746
Class 2	1,823,200	218.5	10847.240
Class 3	3,032,600	218.5	18042.981
Overall	2,360,510	218.5	14044.224

¹ Ansan transport master plan (1998)

² Korea Ministry of Labour (1999)

5.3.5. Initial network performance indices

The bid-rent network equilibrium model requires two sets of initial network performance indices for the simulation; they are initial locational attractiveness and initial transport impedance. Since the solution-finding mechanism for the model is fundamentally heuristic, a good model outcome may depend on a good initial solution. Thus, the initial values of transport costs and locational benefits would be essential.

Locational attractiveness represents activity opportunities available in a specific location. In this study, *the posted price of standard land* was used as an initial locational attraction. The Korea Ministry of Construction and Transportation publicly

announces a standard index for the value of each land lot every year. The evaluation is undertaken by the Korea Association of Property Appraisers. The process is as follows: first, land lots are classified in terms of land-use patterns; and then about 1.5% of the sample that is believed to represent the whole land value is selected; finally, the price with respect to the unit area W/m^2 is evaluated.

Table 5-9 The posted price of standard land (1998)

zone	No. of sample	Average	S. Deviation	Min	Max
1	70	341.2	113.1	225.0	710.0
2	63	354.0	123.9	235.0	850.0
3	35	376.9	128.2	240.0	700.0
4	60	305.8	193.5	14.0	850.0
5	28	424.8	174.9	220.0	900.0
6	30	509.8	357.6	140.0	1,650.0
7	88	247.4	158.6	9.0	880.0
8	60	343.8	171.7	9.5	1,000.0
9	76	296.3	160.2	10.0	740.0
10	26	382.4	230.2	34.0	900.0
11	18	812.8	498.2	210.0	1,800.0
12	17	470.4	445.5	11.0	1,050.0
13	49	637.0	412.5	21.0	1,850.0
14	11	737.3	754.8	310.0	3,000.0
15	13	26.3	4.1	21.0	31.0
16	22	185.1	186.2	15.0	720.0
17	16	270.6	107.3	25.0	420.0
18	56	316.3	219.5	16.0	1,050.0
19	24	514.2	344.8	240.0	1,450.0
20	47	395.4	209.8	149.8	1,027.5
21	74	295.6	165.2	9.3	940.0
22	168	280.9	193.1	10.0	790.0

Table 5-9 shows the posted price of standard land (1998) in Ansan. There are some unsatisfactory aspects. First of all, zones 14 and 15 have small samples. The figures may be underestimated or overestimated. In addition to the number of samples, some zones have large standard deviations. This would mean that households would heterogeneously perceive the zones. Notwithstanding these unsatisfactory aspects, using the posted prices as an initial locational attractiveness does not necessarily matter because the vector of the attractiveness is adjusted in the model run.

The net interaction impedance, which is a composite measure of locational attraction and travel time, is used as a network performance index in the lower level. Since the values of the initial locational attractiveness have already been decided, the remaining task is to select an appropriate travel time for the initial value. In this study, three

candidates were examined; they were the solutions of system optimal, user optimal, and all-or-nothing assignment. The system optimal solution is one extreme that would represent the lower bound performance in the network. In contrast, the outcome of all-or-nothing assignment is the other extreme that would be understood as the upper bound performance of the network. The simulation separately ran with the respective candidates. Then, the final outcome recording the highest average bid-rent was chosen. This is because the situation is believed to represent the level of maximised social welfare that the bid-rent network equilibrium model intends to address; note that the location market has been assumed to be perfectly competitive.

5.4. The design of the simulation

As noted in the introductory section of this chapter, the simulation follows the same strategy as adopted in the study of ISGLUTI (Webster et al., 1988) and the model of MASTER (Mackett, 1992; 1993a). Hence, the experiment for the validation of the model is carried out in both the base and the policy runs. Each run is simulated in the single class and the multiclass bid-rent network equilibrium models.

The base run is defined as a simulation conducted using survey data. The policy run is performed modifying some input data. The changes in the input data are regarded as tests of policies. The results of the model runs should be understood as a demonstrative simulation rather than a definitive forecast. This would be because the availability of data for the application is relatively poor and the experiment has some strong assumptions (Mackett, 1993a). Therefore, the discussion on the outcomes focuses on the ability of the bid-rent network equilibrium model to represent the relationship between transport and the location of activities.

Two scenarios for the policy run were tested; they were the congestion charging run and the greenbelt run. The former introduces increases in transport costs to and from the city centre in spatial interactions. The latter assumed an increase in residential stocks. In both cases, the simulation sought to illustrate changes in model outcomes produced by the policies. The descriptions of the scenarios are given as follows:

The congestion charging run: the city of Ansan has suffered from chronic congestion. In order to reduce car uses in the primary activities of people's daily life, the local council has introduced the charging scheme. The cordon has circled the city centre, which has covered zones one, ten, eleven, and twelve.

The greenbelt run: the population in Ansan has increased by five percent. In order to accommodate the increased number of households, the local government has released green belt area in zones four, seven, and eighteen. The released areas have been used for housing development.

Comparisons between the outcomes of the base and the policy runs can be made using elasticity values. These values are useful for examining the behavioural difference between household classes. In this study, the formulation of the linear arc elasticity is

used, which is in line with the research of ISGLUTI (Webster et al., 1988) and the study of MASTER (Mackett, 1992; 1993a). The value is given by

$$e_m = \frac{(q_m^p - q_m^b) / 0.5(q_m^p + q_m^b)}{(c_m^p - c_m^b) / 0.5(c_m^p + c_m^b)} \quad (5.3)$$

where q_m^p is the class-specific number of spatial interactions in the policy run;
 q_m^b is the equivalent number of the base run;
 c_m^p is the class-specific cost of spatial interactions in the policy run; and
 c_m^b is the equivalent cost in the base run.

The values of elasticity for each policy run are calculated against the results of the base run. The values represent the overall changes in the level of the spatial interaction of each class between the base and the policy runs. The elasticity treated in this section is the direct effect only. When the volume adjustment between classes, which was not represented in this case study, is considered, the calculation of cross elasticity values would be attractive. This is because the cross effect is useful to test the volume shift between user classes by a class-specific policy. In particular, the shift can be useful to investigate an intermodal network as some research focuses on.

In general, the model can produce huge quantities of output. Two principal outputs are noted. First, the model offers a class-specific spatial distribution. The fundamental purpose of this type of model is to describe the allocation of households across an area. The model developed in this study faithfully serves the basic necessity. Hence, as with some other models being used, the results can be easily used for a trip table analysis. The other important output is a network performance index in terms of transport costs and locational benefits. The model produces the endogenously determined travel time and locational attractiveness. The values are determined by the process of interaction between transport and the location of activities. The two principal outputs may make the model a useful tool for policy tests.

5.5. The base run

5.5.1. Introduction

This section presents the results of the base run of the bid-rent network equilibrium model. The base run was defined in the simulation design section of this chapter as a model run using the data surveyed. Hence, the results of the base run mean a changed urban structure from the base year by the simulation. Usually, the base run incorporates a calibration step. That is, the base run involves the process of finding and adjusting values of parameters in order to make model outputs conformable to the model specification, which yields the best fit with the observed data. In the calibration, certain previously defined measures of goodness-of-fit are normally used. However, it was infeasible in the case study to obtain the reference statistics in terms of the time period for the model design. This study has assumed that the bidding game for locations requires a sufficient amount of time to allow all households that consider moving the location of activities to make their decision. It has been also supposed that the output of the base run is regarded as the structure after the end of the bidding competition, namely the urban structure at the end of the game. According to the statistics for the average tenure in residence (2001) in Korea, which is one of the irregular surveys conducted by the Korea National Statistical Office, 6.6 years was reported as the tenure in Ansan. Thus, the results of the model run would be the equivalent urban structure six to seven years after the base year; for discussion details on the time period, see the section on the conceptual basis of the model in chapter three. Unfortunately, since the Ansan transport master plan (1998), the local authority has conducted no comprehensive transport survey that is a similar scale to the master plan. Thus, the base run could not conduct the calibration comparing the model outcomes of the base run with those of real statistics relating to the time period.

In the next section, the results of the model runs are investigated. The description on the results is presented in two ways: first, the general trends of the model outcomes are discussed; secondly, comparison and contrast of the output are provided between the single class and the multiclass bid-rent network equilibrium models.

5.5.2. The results of the base run

Figure 5-4 at the end of this section shows changes in the number of households in zones for the single class bid-rent network equilibrium model, and Figure 5-5 shows those of the multiclass model respectively. The changes in these diagrams mean the difference in the number of households between the initial volumes of households in each zone of the base year and those in the results of the base run. The colour blue represents zones that show the increased number of households, and the colour red shows areas in which the number of households has decreased. Blank areas mean zones showing no significant changes in household numbers.

In general, the two models suggest that there has been an increase in population across zones in the southern belt of the city and a decrease in the number of households in Northeast Ansan. It has been also shown in the diagrams that there are some buffer zones between the southern belt and the Northeast areas. In fact, these buffer zones show no significant changes in the number of households. These results are not surprising. The southern belt is relatively more attractive for residential locations than the other areas for several reasons. First of all, the southern areas have good access to the primary workplace locations. The belt includes the two factory districts and borders on the city centre. Thus, living in the southern belt would save travel time in spatial interactions for primary activities. In addition to this property, the southern zones show the good quality living environment. The zones border on the Yellow Sea. This would mean that the areas do not significantly suffer from extra congestion generated by through traffic and have the good opportunity for refreshment. Another important aspect on the good quality of the living environment is open space. For example, according to the Ansan transport master plan (1998), zone 16 alone has 23.5 percent of the entire open space in the city. These good aspects of the job opportunity and the residential environment are thought to be the reason why households have bid more rent than the other areas. The two good residential attractions together with the bidding have created the increase in the number of households in the southern belt.

Even though the above explanation suggests an intuitive understanding of the outcomes in terms of the spatial distribution of households, the discussion is indirectly related to the model operation. The investigation concerning the operational

mechanism of the model should focus on the examination into the network performance indices of transport costs and locational benefits. In the analysis into the network performance indicators, three representative patterns are found. First of all, as would be expected, the demand for a specific location has increased when the cost has gone down and the attraction has gone up. In contrast, a decrease in the attractiveness and an increase in the impedance have reduced demands for locations. Secondly, when changes of one factor dominate those of the other factor, net changes in terms of demands for locations have been determined by the dominant factor. That is, even though one factor has increased or decreased, the larger responses of the other factor compensated the effect of the factor. Finally, there have been no significant changes in demands for locations that show counterbalancing effects between the cost and the attractiveness. Zones with a similar degree of increased cost and increased attractiveness or with decreased cost and decreased attractiveness have no distinct changes in the locational demand; zone-specific references are found in Table 5-11 for the single class model and in Table 5-12 for the multiclass model respectively.

Responses in the changes of household numbers appear to have a relationship to the size of zones. While most zones that have suggested changes in the number of households show a similar magnitude in the switch, the effect was larger in the geographically small areas; full references are found in Table 5-11 and Table 5-12. The asymmetric patterns would mean that small-sized zones would respond more elastically to changes in the household number than large-sized counterparts.

The discussion on the relationship between zonal sizes and model responses would be simply examined by merging the results of several small zones. Figure 5-3 shows a modified version of the outputs of the base run of the bid-rent network equilibrium model. In this diagram, zones 14, 15, and 17 were merged into the single large zone, and zones five and six were integrated. The groupings would be reasonable in the sense that the merged zones would be thought to share relatively homogeneous characteristics. Zone 14, 15, and 17 are located between the city centre and the factory districts. Zones five and six are adjacent to the city centre.

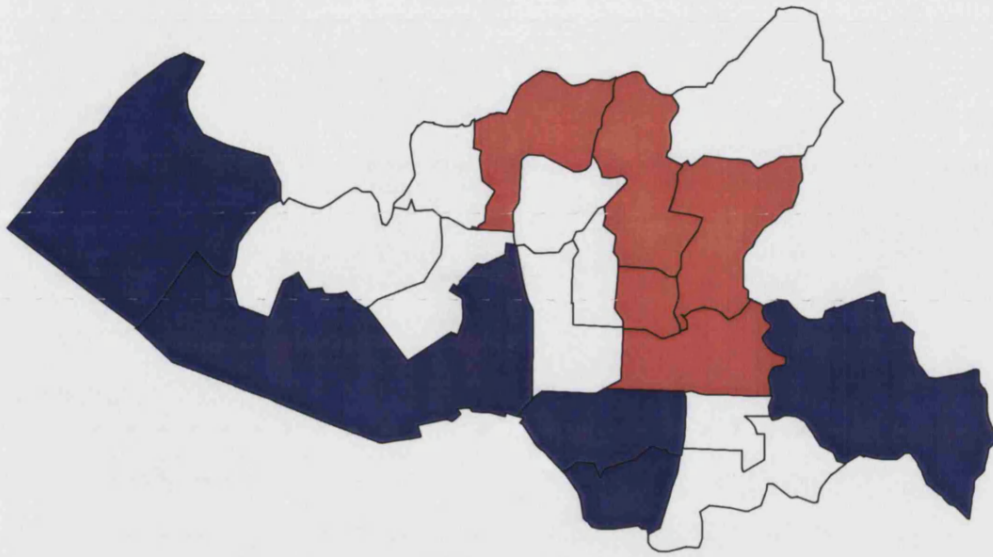


Figure 5-3 The results of the modified base run of the single class model

The output in Figure 5-3 would suggest the mechanism for the relationship between the model responses and the size of zones. First, the effect of responses is likely to be buried in geographically larger zones. This inference can be checked in the merged zones of 14, 15, and 17. Even though zone 14 shows an increase in household numbers, the overall responses in the merged zone suggest no significant changes. Second, an effect of trade-off is another important factor that determines the magnitude of responses in zones. The integrated zones of five and six shows no significant changes in the number of households, even though zone five and six show an increase and decrease of demands respectively. The setoff between the two zones eventually created no significant changes in household numbers. However, this implication is provisional. A confident conclusion requires various scientific references. Experiment simulations in terms of adding or splitting zones would provide evidences to draw a conclusion. However, the investigation in terms of varying the spatial system is a huge field in its own right and in fact beyond the scope of this study. The difficulty is mainly because the spatial opportunity in each zone would have to be redefined. Furthermore, the modification would not be helpful to the model operation because most data available are in a format that satisfies the spatial

system given. Nevertheless, it would be useful to stress that demands for locations might be sensitive to the design of the spatial system.

Several zones show interesting different responses in the changes of population between the two models. Zone three shows an increase in household numbers in the single class model but no considerable changes in the multiclass model, whereas zone ten shows the reverse. While zone six suggests a decrease in household numbers in the single class model but no significant changes in the multiclass model, zone 15 and 21 show the reverse. In fact, these differences are minor compared to the entire responses of the model runs; note that around eighty percent of zones show similar responses between the two models. In addition, there have been no sudden different responses. All the zones mentioned above show a smooth transition between coloured and blank. It is found that no zones changed from red to blue or blue to red. This would mean that the results generated by the two models are relatively stable.

Table 5-10 Social welfare indices of the base run

	Single class	Multiclass
Mean percent changes in the transport impedance:		
Lower income class	N/A	-7.93
Middle income class	N/A	-9.50
Higher income class	N/A	-1.17
Total	-6.31	-8.38
Mean percent changes in the locational attractiveness	17.65	14.87

Table 5-10 shows the mean percentage changes of transport impedance and locational attractiveness in the base run. Since changes in the two factors would determine average costs and benefits in the spatial interaction, the values are interpreted as indices of social welfare in this study. The values are calculated against the initial network performance indicators of the mean travel time and the mean locational price. The figures are weighted averages that consider the size of population in each zone for the locational benefit and in each OD pair for the transport impedance.

As would be expected, both models suggest negative values for transport impedance and positive values for locational attractiveness. These results are thought to be created by the selfish bidding of households for locations. That is, in the process of the competition, people choose locations that reduce costs and increase benefits in the

long run. The behaviour is related to the fundamental assumption for the travellers as economic men who minimise the cost and maximise the benefit.

The multiclass bid-rent network equilibrium model shows the larger responses in the changes of transport costs compared to those of the single class counterpart. This result might mean that the multiclass model would be more sensitive to changes in the transport impedance than the single class model. However, it has not been generally known whether either model is more sensitive than the other. It could be thought that this difference would be related to the parameters that have been used in the multiclass model for representing the class-specific behaviour. If the parameters have been finely calibrated, the two models might produce very similar mean percentage changes in the cost of travel time. The tuning, however, requires considerable data to specify the values. In fact, the job is indirect to the purpose of this study. This research has simply assumed that the given parameters are well identified. A further important point in this issue is the OD matrix. It was not feasible to obtain the class-specific OD tables for simulating the lower level of the multiclass bid-rent network equilibrium model. This study has assumed that the ratios of classes in the volumes of the origin-destination matrix are exactly the same as those of sample household data in the upper level. Hence, the volumes in the OD table might be biased.

Although the bid-rent network equilibrium model shows less sensitive changes in transport costs than the multiclass model, the single class model suggests larger responses in the changes of locational benefits. This result is not surprising. The bid-rent function in the upper level has been formulated to maximise utility with respect to the network performance indices of the transport cost and the locational attractiveness. This is represented by a composite function of the two performance factors. Even though the random component that has been incorporated in this study could open the possibility of the utility being determined inside or outside the deterministic indifference curve, it would be expected that the stochastic indifference curves would be in the vicinity of the deterministic utility level. Hence, both models would produce a similar magnitude of utility value. This can be simply checked in Table 5-10 by examining the net welfare that the two factors produced together in each model; changes in the transport impedance show negative signs in both models, which suggests increases in the benefit from a social viewpoint; then, the total social

benefits created by the model run can be calculated summing the locational benefits with the transport savings; the results are similar, which is in line with the process of the formulation of the model. The different values of the cost and benefit in the two models are due to the solution algorithm. The heuristic algorithm that has been adopted for the solution-finding method in this study produces a mutually consistent solution. Hence, two critical factors of the transport cost and the locational attractiveness are determined in such a way in which they cope with the utility values. Unfortunately, there is no known way to decide that either solution is better than the other. This is one of the most critical issues in bi-level mathematical programmes.

It is worth stressing that the multiclass model suggests class-specific benefits in the changes of transport costs; note that the single class bid-rent network equilibrium model shows the average benefit for the entire people only. Again, in Table 5-10, the lower and middle income classes have more benefits than the higher income households. The benefits are mainly created by the residential changes of people to maximise utility in the long run. That is, the changed demand for locations is the background reason for the different benefits. It is not difficult to suppose that rich households are less keen on saving transport costs by moving their home. In contrast, the poor are likely to be elastic in saving transport costs. The dissimilar behaviour is reflected in the values of the benefits for each class. An interesting result in the sensitive group is that the middle class households are the greatest beneficiaries in terms of saving the transport cost. This would be because the middle class has more reserve power in changing home than the lower income group. It might be expected that the lower income group would change more frequently than the middle income class. However, their choice would be spatially less flexible than the mid group because of budget limits. This means that the middle income class has a more advantageous position to save the transport impedance than the lower income class.

In conclusion, it can be expected from the above discussion that, everything else being equal, there will be an increase in the number of households in the southern belt of Ansan and a decrease in the Northeast of the city. These results are created by the non-cooperative competition of households for occupying residential locations in which households seek to maximise welfare, namely to minimise transport costs and maximise locational benefits. The two models suggest that the competition saves

resources in terms of travel time and increases the worth of locations. The middle income class is forecast to be the maximum beneficiary.

It is important to note that even though the single and multiclass models agree on the general spatial pattern, the zone-specific effects and their causes would be different between the two models. For example, zone 14 shows an increase in the number of households in both models, but their background causes are different. The explanation goes towards an effect of dominance for the single class model and a commonsense pattern for the multiclass model. In the single class model, the larger decrease in the impedance has created a net effect of increase in household numbers even though there has been a decrease in the attractiveness. In contrast, the decreased impedance and increased attractiveness have increased household numbers in the multiclass model; the full references are found in Table 5-11 and Table 5-12. These different circumstances are influenced by the introduction of the multiclass cost function in the multiclass model. The variation created by the multiclass cost function causes user classes to perceive the cost differently at a micro-level. These class-specific perceptions lead to the different behavioural pattern of users. This settles the cost and the attractiveness differently from the outcomes of the single class model. The new settlement, in the end, determines the level of demands in each zone.

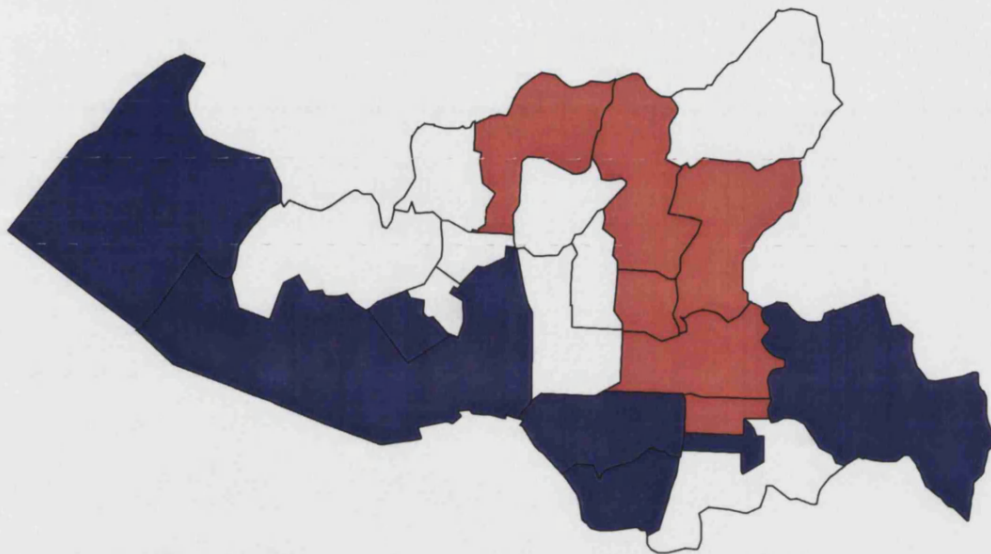


Figure 5-4 The results of the base run of the single class model

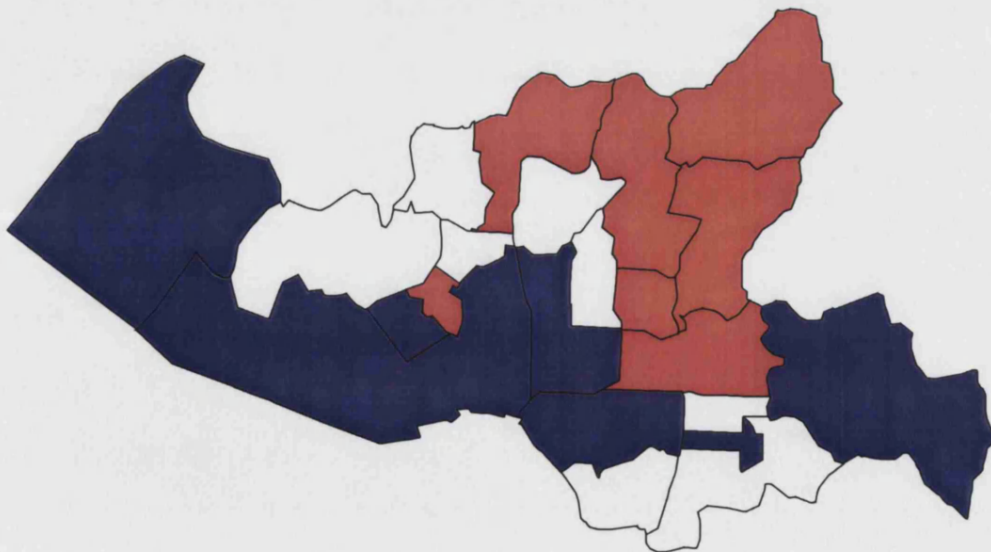


Figure 5-5 The results of the base run of the multiclass model

Table 5-11 Results of the base run of the single class bid-rent network equilibrium model

zone	Impedance ¹		Attractiveness ¹		Flows of spatial interactions							
	before	after	before	after	before	after						
					<i>total</i>	<i>total</i>	<i>m₁</i>	<i>m₂</i>	<i>m₃</i>	<i>m₁</i> (%)	<i>m₂</i> (%)	<i>m₃</i> (%)
1	105.27	108.40	135.42	134.26	11674	11627.08	6697.84	4555.34	373.90	57.61	39.18	3.22
2	84.41	79.10	119.20	143.84	5763	5777.07	2387.85	3078.85	310.37	41.33	53.29	5.37
3	64.80	78.55	145.56	145.17	9698	9719.50	4065.70	5140.88	512.92	41.83	52.89	5.28
4	99.18	108.68	77.96	93.62	11069	11077.46	5047.38	5503.55	526.53	45.56	49.68	4.75
5	98.02	80.00	149.29	139.02	8926	8941.59	3861.69	4624.87	455.03	43.19	51.72	5.09
6	103.40	107.85	108.92	100.43	7628	7613.40	3977.66	3341.63	294.12	52.25	43.89	3.86
7	98.19	114.84	91.40	89.13	6322	6318.61	3073.06	2972.55	273.00	48.64	47.04	4.32
8	118.90	105.44	108.45	82.73	13669	13654.01	6452.92	6583.68	617.40	47.26	48.22	4.52
9	112.50	118.68	70.91	82.54	13105	13113.72	6113.68	6396.28	603.77	46.62	48.78	4.60
10	102.44	104.51	76.34	79.98	9132	9139.89	4231.15	4483.78	424.96	46.29	49.06	4.65
11	116.65	108.73	103.57	93.72	8124	8129.46	3821.37	3939.62	368.47	47.01	48.46	4.53
12	105.39	109.84	111.89	110.41	10119	10092.62	5441.14	4282.61	368.86	53.91	42.43	3.65
13	76.70	86.96	90.60	96.40	8976	8981.54	4224.50	4354.26	402.78	47.04	48.48	4.48
14	108.08	88.14	90.39	78.47	6249	6254.95	2812.87	3139.18	302.90	44.97	50.19	4.84
15	102.66	105.51	140.43	146.58	3849	3848.20	1845.89	1831.81	170.50	47.97	47.60	4.43
16	89.75	75.88	101.73	102.06	6327	6339.05	2702.57	3315.97	320.50	42.63	52.31	5.06
17	80.27	84.63	84.56	87.72	10145	10146.50	4766.21	4917.49	462.80	46.97	48.46	4.56
18	104.56	114.93	76.38	81.07	11890	11890.14	5637.59	5719.60	532.94	47.41	48.10	4.48
19	94.38	115.19	93.31	91.26	16061	16044.58	8026.68	7353.67	664.22	50.03	45.83	4.14
20	117.72	108.65	64.05	82.58	6802	6808.77	3062.54	3420.20	326.04	44.98	50.23	4.79
21	121.10	114.84	91.60	80.33	4033	4034.65	1883.97	1965.72	184.96	46.69	48.72	4.58
22	95.62	80.67	68.06	80.66	4176	4183.21	1814.78	2162.77	205.66	43.38	51.70	4.92
<i>total</i>					193736	193736.00	91949.05	93084.32	8702.63	47.46	48.05	4.49

¹ The value of the zonal average is set as a reference index 100.

Table 5-12 Results of the base run of the multiclass bid-rent network equilibrium model

zone	Impedance ¹		Attractiveness ¹		Flows of spatial interactions							
	before	after	before	after	before	after						
					total	total	m_1	m_2	m_3	$m_1(\%)$	$m_2(\%)$	$m_3(\%)$
1	105.27	102.94	135.42	124.24	11674	11659.51	6678.27	4587.72	393.52	57.28	39.35	3.38
2	84.41	93.81	119.20	138.70	5763	5770.46	2406.63	3077.52	286.31	41.71	53.33	4.96
3	64.80	60.15	145.56	140.24	9698	9692.67	4037.76	5131.48	523.43	41.66	52.94	5.40
4	99.18	102.27	77.96	79.38	11069	11060.80	5069.78	5497.01	494.00	45.84	49.70	4.47
5	98.02	94.87	149.29	148.66	8926	8991.54	3897.98	4620.56	473.00	43.35	51.39	5.26
6	103.40	127.28	108.92	124.86	7628	7631.55	3985.99	3348.55	297.02	52.23	43.88	3.89
7	98.19	135.54	91.40	103.07	6322	6310.11	3051.38	2987.76	270.97	48.36	47.35	4.29
8	118.90	124.44	108.45	95.65	13669	13626.49	6449.79	6559.05	617.65	47.33	48.13	4.53
9	112.50	120.08	70.91	75.39	13105	13088.90	6100.00	6381.96	606.95	46.60	48.76	4.64
10	102.44	103.35	76.34	92.48	9132	9177.63	4269.11	4478.52	430.01	46.52	48.80	4.69
11	116.65	118.34	103.57	106.81	8124	8121.65	3858.45	3900.08	363.12	47.51	48.02	4.47
12	105.39	129.64	111.89	107.67	10119	10101.46	5441.42	4285.97	374.07	53.87	42.43	3.70
13	76.70	82.63	90.60	93.86	8976	8981.03	4236.89	4349.73	394.41	47.18	48.43	4.39
14	108.08	104.03	90.39	98.43	6249	6276.91	2812.01	3163.61	301.29	44.80	50.40	4.80
15	102.66	124.54	140.43	141.87	3849	3836.22	1821.08	1846.54	168.59	47.47	48.13	4.39
16	89.75	80.24	101.73	106.97	6327	6349.94	2697.79	3339.64	312.51	42.49	52.59	4.92
17	80.27	83.67	84.56	89.88	10145	10143.41	4747.91	4919.38	476.11	46.81	48.50	4.69
18	104.56	125.66	76.38	93.74	11890	11895.89	5652.39	5698.90	544.60	47.52	47.91	4.58
19	94.38	135.96	93.31	90.96	16061	16006.00	8013.93	7323.55	668.52	50.07	45.76	4.18
20	117.72	128.23	64.05	95.46	6802	6824.92	3038.11	3462.75	324.05	44.51	50.74	4.75
21	121.10	135.54	91.60	92.07	4033	4005.66	1876.65	1958.94	170.07	46.85	48.90	4.25
22	95.62	104.04	68.06	92.60	4176	4183.24	1805.74	2165.08	212.43	43.17	51.76	5.08
total					193736	193736.00	91949.05	93084.32	8702.63	47.46	48.05	4.49

¹ The value of the zonal average is set as the reference index 100.

5.6. The congestion charging run

5.6.1. Introduction

The congestion charging run is concerned with changing transport costs to and from the city centre in the spatial interactions of households, namely increasing costs of travel for journeys involving the city centre. The cordon line circled the city centre of Ansan, which included zones 1, 10, 11, and 12. The amount levied in this procedure was equivalent a half an hour value of travel time. The charge was imposed as a fixed rate across household classes, but the class-specific perceptions for the extra costs are differentiated in the multiclass model because the model has incorporated the class-specific weighting factor. No discount scheme for certain categories of households in terms of their social status and residential area was considered. In the next section, discussion detailed about the congestion charging run is presented.

5.6.2. The results of the congestion charging run

The single and multiclass bid-rent network equilibrium models generally agree on the spatial distribution of households in the congestion charging run. As shown in Figures 5-6 and 5-7 at the end of this section, fairly similar responses are found in most zones between the two models. Even though a few areas show rather contrary patterns, no drastic disagreement in the demands is captured; note that all the differences are related to a smooth transition from coloured to blank or the reverse. As discussed in the section on the base run, the responses are larger in geographically small areas. The response again would imply that the demands for locations would be sensitive to zonal sizes; the full reference table is suggested in Table 5-18 and Table 5-19.

Table 5-13 Mean percentage changes in the network performance indices for the charging zones in the congestion charging run

Zone	The transport impedance		The locational attractiveness	
	Single class	Multiclass	Single class	Multiclass
1	-3.95	-4.87	-6.01	-5.36
10	-3.32	-3.55	-16.08	-12.25
11	-2.76	-2.67	-13.55	-15.40
12	-4.14	-3.16	-8.27	-9.62

Table 5-13 shows the mean percentage changes in the network performance indices of transport costs and locational benefits in the congestion charging scheme. The values are calculated referenced against those of the base run. As would be expected, the transport cost and the locational attractiveness in the zones inside the cordon line have decreased. These areas include zones 1, 10, 11 and 12. In both models, the impedance changes show a marginal decrease, but the locational benefit shows a relatively larger decrease. The marginal responses in the cost reflect that the charging scheme would not sufficiently relieve the chronic delay in the city centre, even though the scheme would decrease traffic volumes within the charging boundary. This would be because the levied amount would not be enough to resolve the congestion in central Ansan. It might be thought that increasing the amount would create more responsive results, or some trial-and-error experiments in terms of an incremental raising or reducing the collection would offer a desirable charge rate. In fact, these investigations are indirect to the purpose of this chapter. The interesting experiments are left for future study. In spite of this provisional conclusion, it is useful to stress that the two models suggest that congestion level in the charging areas would be reduced by the charging scheme.

An understanding on the decrease in the locational attractiveness is rather complex. Since the charging zones cover the city centre, people may think that these areas are undesirable for residential locations. Thus, people would bid for the areas relatively less than for the other zones. The asymmetric bidding may cause a decrease in the locational attraction. However, this explanation does not satisfactorily offer a systematic interpretation of the operation in the bid-rent network equilibrium model. It would be better to understand that the charging scheme has made these areas less attractive. This is inferred from the fact that the scheme requires more costs than those previously required for the spatial interaction. A technical explanation for the decrease should focus on changes in household numbers. In these areas, the number of households has decreased, which has meant that the number of bidders has decreased; therefore, the maximum expected bid-rent is likely to decrease in the calculation of logsum. The mechanism leads to a decrease in the locational attractiveness.

In summary, the congestion charging scheme has decreased the impedance and the attractiveness of the zones inside the boundary. The effect was larger in the locational benefit than the transport cost. These asymmetric responses cause a decrease in

household numbers in the central areas. It is worth noting that there have been no significant differences in the responses between the two models, as shown in Table 5-13. Again, there is no established belief on either outcome should be more sensitive than the other. However, it should be noted that similar responses would be influenced by an aggregation process; the values are calculated for zonal average without considering class-specific behaviour nor zone-to-zone responses. These two aggregation biases may prevent the outcomes of the models from being differentiated.

Table 5-14 Mean percentage changes in the network performance indices for the adjacent charging zones in the congestion charging run

Zone	The transport impedance		The locational attractiveness	
	Single class	Multiclass	Single class	Multiclass
North				
7	7.97	5.90	2.78	4.20
8	14.86	12.15	2.22	2.14
9	8.46	15.57	4.59	3.99
South				
2	3.63	0.15	3.71	7.62
6	3.67	4.93	5.54	3.99
16	2.93	5.18	7.69	15.21
20	0.19	0.02	8.54	7.35

Table 5-14 shows changes in the network performance indicators for the zones adjacent in the charging areas. As would be expected, the impedance and the attractiveness have increased in both single and multiclass models. Again, fairly similar responses are observed between the two models. This would be also because of the aggregation processes explained above. Two important patterns are observed in Table 5-14. The zones that are geographically southern adjacent to the cordon were predicted to show a marginal increase in the cost but a relatively larger increase in the attraction. In contrast, the northern zones adjacent to the charging boundary were forecast to show a marginal increase in the benefit but a larger increase in the impedance. These trends appear to be influenced by through traffic. The southern zones border on the Yellow Sea, and so the zones have not much through traffic. This causes that the areas were marginally affected by the charging scheme. In contrast, the northern counterparts were significantly affected because the areas have a large amount of through traffic. The converse effects have determined the different patterns of demands for locations between the North and South. Namely, the zones in the South have shown increased demands while the areas in the North the reverse.

Table 5-15 Mean percentage changes in the network performance indices for some interesting zones in the congestion charging run

Zone	The transport impedance		The locational attractiveness	
	Single class	Multiclass	Single class	Multiclass
17	13.76	19.42	-4.61	-4.78
18	12.61	16.23	-5.95	-2.98
22	-16.12	-9.81	10.22	8.26

Some zones show interesting responses to the congestion charging scheme. As shown in Table 5-15, zones 17 and 18 show larger changes in the transport impedance. This appears to be resulted from the fact that a vertical corridor, which is located next to the main north corridor and leading to the charging areas, passes through these zones. A large amount of traffic that seeks bypasses has converged in these zones. Hence, the cost of travel has considerably gone up. The cost increase in turn has decreased the number of households in the zones. The decreased household numbers have reduced the attractiveness. The reduction of the locational attraction, technically, results from the mechanism of the calculation for logsum value. In zone 22, larger responses in both the transport cost and the locational benefit are observed. The cost decrease is because the main horizontal corridor to the city centre passes through this zone. Traffic that used to use this corridor seeks bypass in order not to have additional costs. This has drastically reduced the cost. The increase in the attractiveness results from the increased number of households. The charging scheme has motivated people to live close to the locations of activities. As mentioned above, the main factory district is located in this zone. This opportunity has attracted households to move in this area. The increased household numbers have increased the maximum bid-rent.

Table 5-16 Social welfare indices for the congestion charging run

	Single class	Multiclass
Mean percent changes in the transport impedance:		
Lower income class	N/A	0.54
Middle income class	N/A	-1.50
Higher income class	N/A	-2.02
Total	-0.51	-0.66
Mean percent changes in the locational attractiveness	-0.34	-0.32

Table 5-16 shows public welfare changes of the congestion charging scheme. The direct effect of the scheme was a saving of transport costs in both models, though the changes were marginal. The marginal variation is mainly because the delay caused by

the extra congestion adjacent to the charging zones balances the extra cost reduction inside the city centre. However, it should be borne in mind that the responses might be influenced by the level of charging. As noted above, the trial-and-error investigation could be an interesting sensitivity experiment for future study.

Table 5-17 Elasticity of cost changes to changes of residential locations in the congestion charging run

	Single class	Multiclass
Changes of residential locations:		
Lower income class	N/A	-0.64
Middle income class	N/A	-0.61
Higher income class	N/A	-0.03
Total	-0.43	-0.21

The interclass effects can be examined in the results of the multiclass bid-rent network equilibrium model. As shown in Table 5-16, the maximum beneficiary from the charging scheme was the higher income class and the poorest added the costs. The result would be because the perceived extra costs in the scheme are differentiated to the household classes. It is not difficult to imagine that the rich travel with marginal impedance imposed by the scheme. However, the amount added would be significant to the lower income class. This explanation is supported by examining the elasticity values of cost changes to changes of residential locations. As shown in Table 5-17, the frequency with which the higher income class moved home was not effectively affected by the congestion charging scheme, but the effects were larger for the middle and lower income classes. Overall, the two models suggest that the maximum beneficiary from the congestion charging scheme is the higher income group.

It is worthwhile commenting on the different viewpoints on the charging scheme between an individual and a planner. From an individual viewpoint, the scheme is acceptable only if the value of saved travel time surpasses the money paid. Thus, it might be argued that even though the congestion charging scheme would reduce the total transport costs in the area, the scheme should be implemented in the case of an individual not suffering losses from the scheme. In contrast, the loss of an individual is not important from the social viewpoint. This is because the amount of money paid does not disappear but is transferred from an individual to a local authority. Thus, the problem on whether the scheme would reduce the overall social costs is meaningful

from the social perspective. The discussions in this chapter are implicitly based on the social viewpoint as normally assumed in model-based policy interpretations. In this context, the bid-rent network equilibrium model and this case study do not explicitly consider the money balance of an individual.

In conclusion, the effect of the congestion charging scheme in the charging zones was a reduction in delay. However, the scheme created a larger decrease in the locational attractiveness because the scenario requires extra costs. Consequently, the number of households inside the boundary has decreased. The zones adjacent to the charging areas showed two broad patterns. While the number of households has increased in the South, the number has decreased in the North. These converse results were understood in terms of the effect of through traffic. A large amount of through traffic in the North has considerably increased the transport cost, but less through traffic in the South has contributed to the marginal increase in the impedance. It is worth noting that the maximum beneficiary from the scheme was the higher income class, but there have been no significant welfare improvements across the classes. Of course, the level of social welfare might depend on the amount of the charge. Thus, some trial-and-error experiments in terms of raising or reducing the levy would suggest a desirable rate of charge with respect to an improvement of social welfare as well as the reduction of congestion. These issues are left for future study.

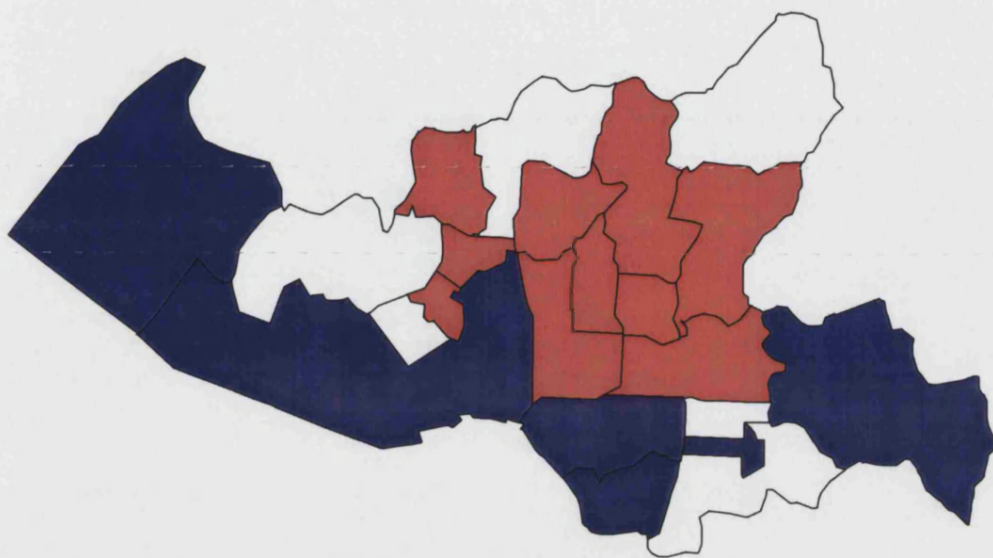


Figure 5-6 The results of the congestion charging run of the single class model

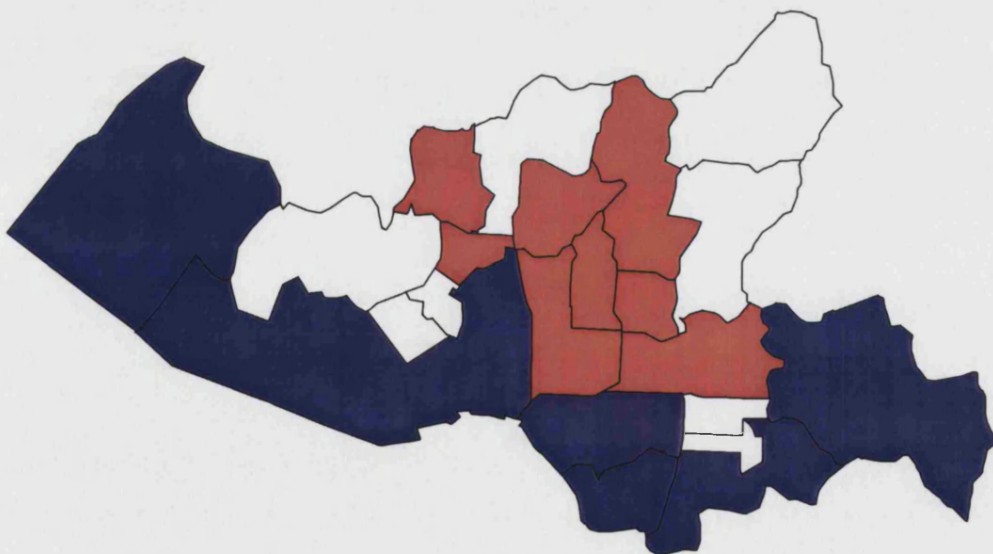


Figure 5-7 The results of the congestion charging run of the multiclass model

Table 5-18 Results of the congestion charging run of the single class bid-rent network equilibrium model

zone	Impedance ¹		Attractiveness ¹		Flows of spatial interactions							
	base	policy	base	policy	base <i>total</i>	policy						
						<i>total</i>	<i>m₁</i>	<i>m₂</i>	<i>m₃</i>	<i>m₁</i> (%)	<i>m₂</i> (%)	<i>m₃</i> (%)
1 ²	108.40	104.12	134.26	126.19	11627.08	11614.68	5670.93	5451.96	501.79	48.78	46.90	4.32
2 ³	79.10	81.97	143.84	149.19	5777.07	5789.11	2475.29	3017.09	296.74	42.76	52.12	5.13
3	78.55	74.85	145.17	145.68	9719.50	9737.49	4160.13	5077.64	499.72	42.72	52.15	5.13
4	108.68	105.72	93.62	88.83	11077.46	11087.29	5305.99	5277.97	491.33	47.91	47.66	4.44
5	80.00	74.12	139.02	147.39	8941.59	8963.26	3826.14	4676.72	460.40	42.69	52.18	5.14
6	107.85	111.80	100.43	105.98	7613.40	7619.22	3655.28	3619.47	336.46	48.02	47.55	4.42
7 ³	114.84	123.99	89.13	91.61	6318.61	6311.49	3078.62	2963.54	272.33	48.75	46.93	4.31
8 ³	105.44	121.11	82.73	84.56	13670.01	13654.05	6650.21	6415.33	591.51	48.69	46.97	4.33
9	118.68	128.72	82.54	86.33	13109.72	13092.39	6282.87	6240.42	579.10	47.95	47.63	4.42
10 ²	104.51	101.04	79.98	67.12	9135.89	9126.14	4395.59	4328.88	401.67	48.16	47.43	4.40
11 ²	108.73	105.73	93.72	81.03	8125.46	8117.11	3898.61	3862.44	359.07	48.01	47.57	4.42
12 ²	109.84	105.30	110.41	101.28	10092.61	10081.49	5010.81	4659.44	424.24	49.64	46.16	4.20
13	86.96	78.88	96.40	88.69	8977.54	8984.69	4186.67	4379.98	413.03	46.62	48.78	4.60
14	88.14	81.17	78.47	74.10	6254.95	6260.00	2980.56	3001.03	279.40	47.61	47.93	4.46
15	105.51	100.82	146.58	135.54	3848.20	3837.99	1949.77	1733.01	155.21	50.80	45.15	4.04
16 ³	75.88	78.10	102.06	109.91	6339.05	6357.30	2691.54	3335.48	330.28	42.34	52.47	5.20
17	84.63	96.27	87.72	83.68	10146.50	10125.03	5062.84	4642.91	419.28	50.00	45.86	4.14
18	114.93	129.42	81.07	76.24	11890.14	11878.40	5861.62	5514.03	502.74	49.35	46.42	4.23
19	115.19	123.88	91.26	97.68	16044.58	16050.44	7970.53	7391.60	670.31	49.72	46.10	4.18
20 ³	108.65	108.85	82.58	89.63	6808.77	6817.02	3090.78	3400.15	326.09	45.34	49.88	4.78
21	114.84	119.40	80.33	83.82	4034.65	4037.38	1960.72	1901.49	175.17	48.56	47.10	4.34
22	80.67	67.66	80.66	88.91	4183.21	4194.05	1783.55	2193.75	216.75	42.53	52.31	5.17
<i>total</i>					193736.00	193736.00	91949.05	93084.32	8702.63	47.46	48.05	4.49

¹ The value of the zonal average is set as the reference index 100.² Zones inside the cordon line.³ Zones adjacent to the cordon line.

Table 5-19 Results of the congestion charging run of the multiclass bid-rent network equilibrium model

zone	Impedance ¹		Attractiveness ¹		Flows of spatial interactions							
	base	policy	base	policy	base	policy						
					<i>total</i>	<i>total</i>	<i>m₁</i>	<i>m₂</i>	<i>m₃</i>	<i>m₁</i> (%)	<i>m₂</i> (%)	<i>m₃</i> (%)
1 ²	102.94	97.93	124.24	117.58	11679.34	11654.09	5648.70	5497.34	508.05	48.47	47.17	4.36
2 ³	93.81	93.95	138.70	149.28	5737.51	5787.86	2475.75	3015.64	296.47	42.77	52.10	5.12
3	60.15	55.46	140.24	138.77	9700.76	9739.86	4161.58	5078.59	499.69	42.73	52.14	5.13
4	102.27	108.38	79.38	87.04	11052.83	11068.91	5293.35	5283.45	492.10	47.82	47.73	4.45
5	94.87	95.15	148.66	146.51	8991.60	8994.33	3839.46	4690.30	461.56	42.70	52.16	5.13
6	127.28	133.55	124.86	129.85	7631.50	7632.20	3653.69	3635.22	338.29	47.90	47.66	4.44
7 ³	135.54	143.53	103.07	107.40	6310.10	6309.98	3090.36	2949.21	270.42	48.98	46.74	4.29
8 ³	124.44	139.57	95.65	97.70	13626.50	13607.70	6612.56	6404.24	590.90	48.59	47.06	4.34
9	120.08	138.77	75.39	78.40	13088.92	13076.28	6286.05	6214.23	576.00	48.07	47.52	4.40
10 ²	103.35	99.68	92.48	81.15	9177.65	9129.50	4388.97	4337.79	402.73	48.07	47.51	4.41
11 ²	118.34	115.17	106.81	90.36	8154.65	8123.41	3889.12	3873.70	360.60	47.88	47.69	4.44
12 ²	129.64	125.55	107.67	97.31	10101.36	10088.49	4992.16	4670.42	425.91	49.48	46.29	4.22
13	82.63	87.65	93.86	101.89	8981.03	8986.20	4173.23	4397.16	415.81	46.44	48.93	4.63
14	104.03	102.60	98.43	97.20	6276.93	6272.69	2976.50	2994.52	278.68	47.63	47.91	4.46
15	124.54	120.57	141.87	136.90	3836.22	3839.61	1945.63	1738.14	155.84	50.67	45.27	4.06
16 ³	80.24	84.39	106.97	123.25	6269.99	6356.77	2685.26	3340.40	331.10	42.24	52.55	5.21
17	83.67	99.92	89.88	85.58	10173.42	10123.78	5082.86	4624.28	416.64	50.21	45.68	4.12
18	125.66	146.05	93.74	90.95	11925.88	11871.18	5893.32	5479.93	497.93	49.64	46.16	4.19
19	135.96	138.79	90.96	94.53	16005.93	16020.72	8016.72	7349.73	664.27	50.01	45.85	4.14
20 ³	128.23	128.26	95.46	102.48	6824.95	6832.36	3091.58	3413.18	327.60	45.25	49.96	4.79
21	135.54	137.53	92.07	94.64	4005.67	4008.86	1967.79	1888.60	173.47	48.83	46.87	4.30
22	104.04	93.83	92.60	100.24	4183.27	4211.21	1784.41	2208.24	218.56	42.37	52.44	5.19
<i>total</i>					193736.00	193736.00	91949.05	93084.32	8702.63	47.46	48.05	4.49

¹ The value of the zonal average is set as the reference index 100.² Zones inside the cordon line.³ Zones adjacent to the cordon line.

5.7. The greenbelt run

5.7.1. Introduction

The greenbelt run is concerned with examining transport and land-use implications provoked by the exogenous changes in patterns of land-use. The scenario implemented in this study was an increase in residential stocks in some zones. As described in the section on the simulation design, the population of Ansan was assumed to increase, and some open space in zones 4, 7, and 18 were supposed to be released to accommodate the increased numbers of population. The other land-uses, except residential areas and the former open space that was released, were assumed to be unchanged. It was also supposed that no additional transport infrastructure was supplied. The next section presents the details on the results of the greenbelt run.

5.7.2. The results of the greenbelt run

As shown in Figure 5-8 and Figure 5-9 at the end of this section, the single and multiclass bid-rent network equilibrium models predicted fairly similar responses in the changes to the number of households in each zone by the greenbelt run. The increase in the number of households is observed in the released greenbelt zones. On the other hand, the decrease in household numbers is found at the gateway zones which are located in the way from the released greenbelt zones to either the city centre or the main factory districts. Again, the responses were larger in geographically small zones; the full reference is found in Tables 5-24 and 5-25 for the single class and the multiclass bid-rent network equilibrium models respectively.

Table 5-20 Mean percentage changes in the network performance indices for the released greenbelt zones in the greenbelt run

Zone	The transport impedance		The locational attractiveness	
	Single class	Multiclass	Single class	Multiclass
4	9.94	11.84	12.78	12.58
7	7.14	8.11	13.92	14.76
18	8.23	9.86	16.76	15.02

Table 5-20 shows mean percentage changes in transport costs and locational benefits for the released greenbelt zones in the greenbelt run. These values were calculated referenced against the results of the base run. As would be expected, the network

performance indices have increased in these zones. While the two indicators show a relatively high increase, the responses were larger in the changes of the locational attractiveness. Because of these different effects, the number of households in the released greenbelt areas has increased.

The reason for the increase in the transport cost is self-evident. The simulation has assumed that the population in Ansan had increased with no additional transport infrastructure supplied. Hence, the increased flows of spatial interactions would increase the transport impedance. In contrast, an explanation on the increased attractiveness is rather complicated. The release of greenbelt would imply a redevelopment, which normally produces a good quality living environment. Thus, people would bid higher rent than those before the scheme. Again, a technical understanding in terms of the model operation focuses on the increased number of households and residential stocks. The more people, the higher the aggregate amount of the attractiveness. This is, technically, because of the calculation mechanism of logsum value. It is worth noting that there have been no significant differences between the results of the two models. The similar responses again would be because of the aggregation effects that were discussed in the previous section.

Table 5-21 Mean percent changes in the network performance indices for the gateway zones in the greenbelt run

Zone	The transport impedance		The locational attractiveness	
	Single class	Multiclass	Single class	Multiclass
From 4:				
2	2.11	6.65	-1.06	-1.45
3	7.08	8.83	-2.13	-0.73
5	6.35	4.51	1.30	-1.26
6	6.81	7.28	-0.05	1.88
From 7:				
8	10.23	8.14	-3.19	-1.06
From 18:				
9	7.18	2.24	-2.28	3.96
13	6.03	10.92	-2.56	5.52
14	6.14	12.08	3.03	-3.07
15	7.47	7.25	-2.42	-3.51
17	9.16	12.25	-2.33	1.09

Table 5-21 shows mean percentage changes in the network performance indices of the gateway zones in the greenbelt run. The gateway zones represent areas located between the released greenbelt zones and primary workplace locations. The primary

workplace areas meant either the city centre or the main factory districts. As shown in Table 5-21, the transport impedance shows relatively larger responses while the locational attractiveness shows no significant changes. These asymmetric changes have resulted in the decrease of household numbers in these zones. The responses can be easily understood. The released greenbelt increased the number of households in the released zones. The augmented household numbers meant the increased traffic volumes. The volumes would pass the gateway zones to the primary workplace locations. Thus, roads in the gateway zones would accommodate more traffic with no additional transport infrastructure supplied. The increased traffic would generate extra congestion in the gateway areas. Thus, the cost has gone up.

Table 5-22 Social welfare indices of the greenbelt run

	Single class	Multiclass
Mean percent changes in the transport impedance:		
Lower income class	N/A	3.95
Middle income class	N/A	5.52
Higher income class	N/A	9.94
Total	4.34	4.84
Mean percent changes in the locational attractiveness	3.12	3.98

Table 5-23 Elasticity of attractiveness changes to changes of residential locations in the greenbelt run

	Single class	Multiclass
Changes of residential locations:		
Lower income class	N/A	0.61
Middle income class	N/A	0.75
Higher income class	N/A	0.23
Total	0.57	0.46

Table 5-22 shows social welfare changes in the greenbelt run. The indices of social welfare in this study meant the changes in the network performance indicators of transport costs and locational benefits. As would be expected, the direct effect was an increase in the locational opportunity, but the changes were rather insignificant. This is because the decreased attractiveness of the gateway zones has offset the increased locational attraction of the released greenbelt zones. In other words, even though the effect of the release of open space would be positive to increase the locational opportunity, the overall responses across the city are not distinct because the gateway zones became less attractive owing to the additional congestion. This explanation is supported by examining changes in the transport impedance of the study area; it is

observed in Table 5-22 that the transport cost shows relatively larger increases than the locational attractiveness. Thus, the release of greenbelt area in this scenario would reduce the overall social welfare. However, these results are not definitive because the welfare changes would depend on the amount of released area and its spatial location. In this context, the welfare responses generated by the changes in the size and location of greenbelt release would be another interesting issue for future study.

The class-specific responses in terms of the changes in the transport impedance can be examined using the results of the multiclass bid-rent network equilibrium model; again, note that the single class model only shows overall responses regardless of the class-specific behaviour. The reference for the class-specific responses is shown in Table 5-22. While all the classes were predicted to add extra transport costs, the higher income group was forecast to suffer the most loss in the greenbelt run. The outcome comes up to the expectations of the design for this simulation in the sense that the higher income class was associated with the greatest value of travel time in the multiclass model. In spite of this result, the elasticity values in Table 5-23 show the higher income group is insensitive to the effect of changes in the locational attractiveness. The insensitivity is because the perceived extra costs created by the greenbelt run are not very significant to the higher income class. Thus, it can be summarised that even though the higher income class was predicted to suffer from the greatest loss in the greenbelt run, the scheme designed in this application was not enough to attract the higher income households to elastically move their residence.

In conclusion, the greenbelt run that is related to exogenous land-use changes has created interesting responses in the interaction between transport and the location of activities. Two important issues should be noted. First of all, the values of the network performance indices have increased in the greenbelt released zones. The increased traffic volumes generated by the increased population with no additional transport infrastructure contributed the increase in the transport impedance. The increase in the locational attractiveness was understood with respect to the bidding mechanism accompanied with the increased number of households; the more people, the higher the maximum expected bid-rent. Secondly, the gateway zones from the released greenbelt areas to the primary workplace locations showed an increase in the transport cost, while no significant changes were found in the locational benefit. This response

is self-evident because the increased households create extra congestion in the gateway zones. Even though the release of greenbelt has created dynamic responses in terms of land-use transport interactions, it was found that the simulation designed in this study did not improve social welfare. This is mainly because the increased locational benefits in the released greenbelt area balance the decreased attractiveness created by the extra congestion in the gateway zones. However, this result would not be definitive because the outcome would depend on variations from the strategy for the release of greenbelt areas, namely the result would be sensitive to the amount and geographical locations in the release of open space.

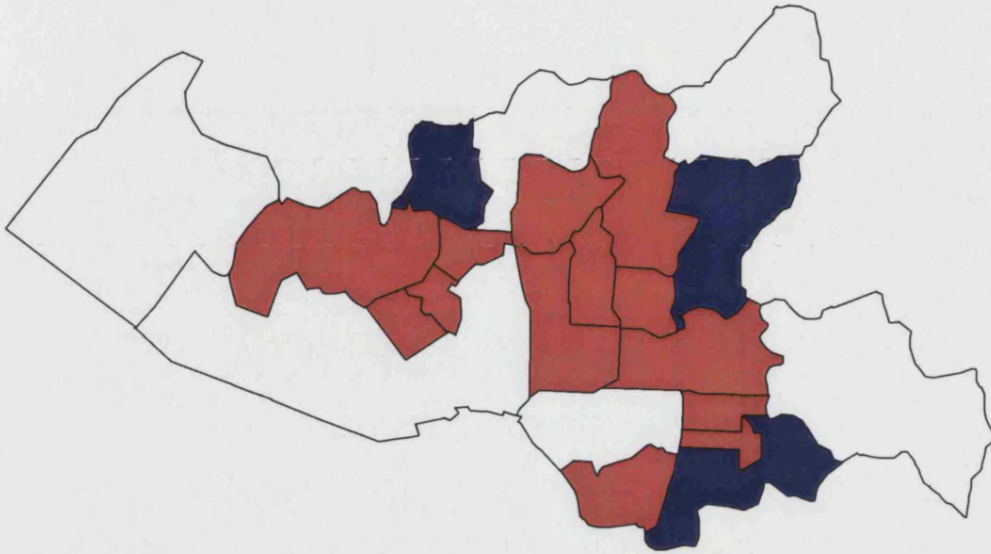


Figure 5-8 The results of the greenbelt run of the single class model

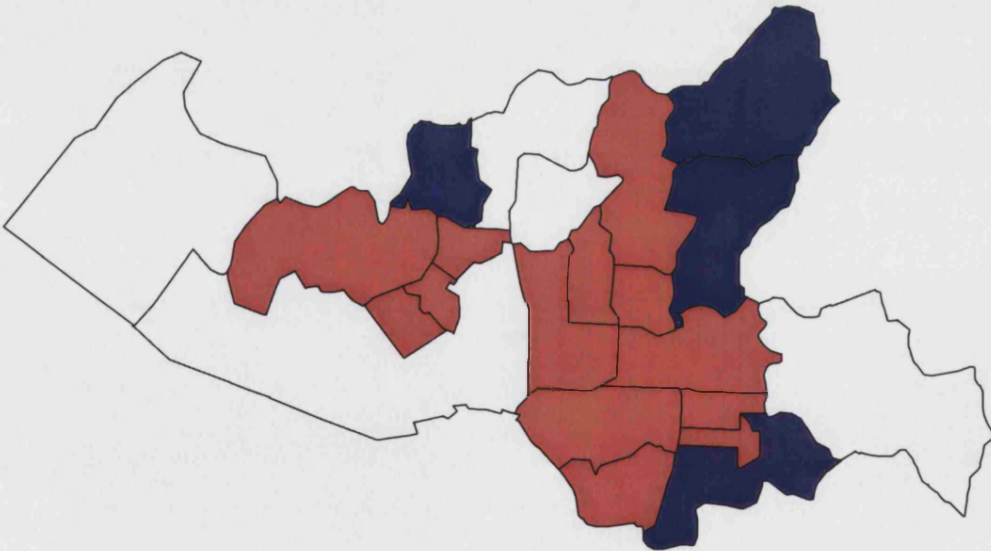


Figure 5-9 The results of the greenbelt run of the multiclass model

Table 5-24 Results of the greenbelt run of the single class bid-rent network equilibrium model

zone	Impedance ¹		Attractiveness ¹		Flows of spatial interactions							
	base	policy	base	policy	base	policy						
					total	total	m_1	m_2	m_3	$m_1(\%)$	$m_2(\%)$	$m_3(\%)$
1	108.40	114.95	134.26	131.54	11627.08	11597.06	6657.09	4553.47	386.50	57.40	39.26	3.33
2	79.10	80.76	143.84	142.33	5777.07	5779.62	2378.57	3091.40	309.66	41.15	53.49	5.36
3	78.55	84.11	145.17	142.08	9719.50	9693.12	4049.93	5131.54	511.65	41.78	52.94	5.28
4 ²	108.68	119.48	93.62	105.58	11077.46	16007.24	7294.65	7965.38	757.21	45.57	49.76	4.73
5	80.00	85.08	139.02	140.83	8941.59	8913.56	3845.02	4618.23	450.32	43.14	51.81	5.05
6	107.85	115.19	100.43	100.38	7613.40	7603.96	3964.78	3337.78	301.41	52.14	43.90	3.96
7 ²	114.84	123.03	89.13	101.54	6318.61	9131.00	4491.70	4279.21	390.09	49.19	46.86	4.27
8	105.44	116.23	82.73	80.09	13670.01	13645.71	6450.94	6575.07	619.70	47.27	48.18	4.54
9	118.68	127.21	82.54	80.66	13109.72	13086.35	6101.50	6378.44	606.41	46.62	48.74	4.63
10	104.51	109.56	79.98	77.31	9135.89	9122.21	4212.17	4478.23	431.81	46.17	49.09	4.73
11	108.73	115.12	93.72	90.43	8125.46	8101.40	3805.24	3923.34	372.82	46.97	48.43	4.60
12	109.84	120.36	110.41	107.50	10092.61	10066.19	5394.37	4298.40	373.41	53.59	42.70	3.71
13	86.96	92.20	96.40	93.93	8977.54	8961.39	4219.73	4346.76	394.90	47.09	48.51	4.41
14	88.14	93.55	78.47	80.85	6254.95	6246.63	2810.01	3131.40	305.22	44.98	50.13	4.89
15	105.51	113.40	146.58	143.04	3848.20	3810.84	1823.29	1823.75	163.81	47.84	47.86	4.30
16	75.88	79.61	102.06	107.62	6339.05	6333.80	2689.02	3306.33	322.45	42.46	52.20	5.09
17	84.63	92.38	87.72	85.67	10146.50	10101.05	4757.31	4891.45	452.29	47.10	48.43	4.48
18 ²	114.93	124.40	81.07	94.65	11890.14	17078.18	8161.55	8174.16	749.47	47.79	47.86	4.39
19	115.19	125.11	91.26	99.84	16044.58	16029.67	8037.75	7306.05	663.86	50.14	45.58	4.14
20	108.65	106.02	82.58	80.23	6808.77	6802.77	3057.53	3416.76	326.48	44.95	50.23	4.80
21	114.84	119.55	80.33	85.05	4034.65	4036.93	1900.99	1951.05	184.89	47.09	48.33	4.58
22	80.67	84.72	80.66	86.85	4183.21	4180.16	1822.47	2156.37	194.33	43.60	51.59	4.65
total					193736.00	206328.84	97925.60	99134.56	9268.68	47.46	48.05	4.49

¹ The value of the zonal average is set as the reference index 100.² The zones that have the released greenbelt areas.

Table 5-25 Results of the greenbelt run of the multiclass bid-rent network equilibrium model

zone	Impedance ¹		Attractiveness ¹		Flows of spatial interactions							
	base	policy	base	policy	base	policy						
					total	total	m_1	m_2	m_3	$m_1(\%)$	$m_2(\%)$	$m_3(\%)$
1	102.94	124.24	113.93	128.76	11679.34	11597.50	6676.22	4554.42	366.87	57.57	39.27	3.16
2	93.81	138.70	100.05	136.69	5737.51	5730.98	2351.46	3073.28	306.23	41.03	53.63	5.34
3	60.15	140.24	65.46	139.22	9700.76	9688.19	4071.37	5105.22	511.60	42.02	52.70	5.28
4 ²	102.27	79.38	114.38	89.36	11052.83	16019.47	7293.82	7962.61	763.04	45.53	49.71	4.76
5	94.87	148.66	99.15	146.79	8991.60	8980.11	3869.78	4617.53	492.80	43.09	51.42	5.49
6	127.28	124.86	136.55	127.22	7631.50	7618.65	4007.86	3330.46	280.34	52.61	43.71	3.68
7 ²	135.54	103.07	146.53	118.29	6310.10	9127.42	4474.15	4263.09	390.18	49.02	46.71	4.27
8	124.44	95.65	134.57	94.64	13626.50	13602.71	6441.47	6579.34	581.90	47.35	48.37	4.28
9	120.08	75.39	122.77	78.37	13088.92	13094.84	6129.96	6343.76	621.12	46.81	48.44	4.74
10	103.35	92.48	109.68	93.42	9177.65	9118.75	4192.26	4520.33	406.16	45.97	49.57	4.45
11	118.34	106.81	127.17	104.87	8154.65	8140.36	3789.00	3982.96	368.41	46.55	48.93	4.53
12	129.64	107.67	140.55	105.12	10101.36	10089.35	5379.06	4292.90	417.38	53.31	42.55	4.14
13	82.63	93.86	91.65	99.04	8981.03	8948.42	4177.39	4355.41	415.62	46.68	48.67	4.64
14	104.03	98.43	116.60	95.41	6276.93	6243.26	2786.58	3144.08	312.59	44.63	50.36	5.01
15	124.54	141.87	133.57	136.89	3836.22	3805.48	1821.01	1807.82	176.65	47.85	47.51	4.64
16	80.24	106.97	86.19	97.96	6269.99	6264.53	2669.55	3297.42	297.57	42.61	52.64	4.75
17	83.67	89.88	93.92	90.86	10173.42	10112.59	4763.76	4892.27	456.56	47.11	48.38	4.51
18 ²	125.66	93.74	138.05	107.82	11925.88	17102.07	8172.39	8170.69	758.99	47.79	47.78	4.44
19	135.96	90.96	145.79	98.18	16005.93	15993.03	8082.78	7298.25	612.01	50.54	45.63	3.83
20	128.23	95.46	131.26	98.45	6824.95	6830.70	3094.97	3442.41	293.32	45.31	50.40	4.29
21	135.54	92.07	137.53	99.06	4005.67	4040.68	1876.95	1958.79	204.94	46.45	48.48	5.07
22	104.04	92.60	111.83	100.59	4183.27	4179.75	1803.83	2141.52	234.40	43.16	51.24	5.61
total					193736.00	206328.84	97925.60	99134.56	9268.68	47.46	48.05	4.49

¹ The value of the zonal average is set as the reference index 100.² The zones that have the released greenbelt areas.

5.8. Comparative behaviour of the two models

This section presents the comparative behaviour of the single and multiclass bid-rent network equilibrium models in terms of their model responses in the simulation. The modelling structure of the two models is similar. Thus, the two models were expected to suggest consistent simulation results; the overall similarities in the model output between the two models were checked in the description of the results from the models. However, the specific outcomes from the two models were rather different. The differences in the outputs arise from the use of the class-specific cost function in the multiclass bid-rent network equilibrium model. As discussed above, the use of the multiclass cost function means that the multiclass model specifically represents the behaviour of user classes when the model evaluates transport costs. The class-specific representation makes the land-use transport interaction differently from that of the single class model. This determines dissimilar values of the transport impedance and the locational attractiveness across the study area. The redefined network performance indicators cause different patterns of the bidding process from that of the single class model. This in the end determines the distinct urban structure that is unlike to the results of the single class bid-rent network equilibrium model.

Drawing general conclusions about the comparative behaviour between the two models requires vast empirical references sufficient to be confident. Unfortunately, there is insufficient data available in a format that permits the empirical validation of this type of model. In fact, the investigation in terms of the comparative responses is beyond the scope of this study. Thus, this comparison should be understood as a provisional examination rather than a definitive conclusion.

The two models generally agreed on the responses in the distribution of households. In the base run, both models suggested that there would be an increase of household numbers in the southern belt of Ansan and a decrease in the Northeast. This output was thought to be created by the non-cooperative bidding game of households for locations. The policy runs have created dynamic responses in the network performance indices. As would be expected, the results of the congestion charging scheme showed a reduction of delay inside the cordon and extra congestion adjacent to the charging zones. These results, which are contrary to each other between the

areas inside and outside the charging boundary, have suggested no significant increase in public welfare from the scheme, but the higher income class was the maximum beneficiary. The finding would be contrary to the common belief that the overall costs in terms of travel time could be saved by the charging scheme. Thus, it would be argued that the road pricing policy would improve the benefits of rich people only rather than the general social welfare. The greenbelt run was concerned with exogenous land-use changes. The scheme would improve the locational opportunity, but the gateway zones became less attractive because of extra congestion. Thus, the overall improvement in terms of social welfare was marginal.

Even though the two models suggested a similar degree of responsiveness in terms of the interaction between transport and the location of activities, the changes were slightly larger in the multiclass bid-rent network equilibrium model. This result would mean that the multiclass model could be more elastic to the model runs than the single class model. Unfortunately, there is no popular sense in which either model is more responsive than the other. The outcomes of this type of model are normally strongly influenced by the study area and the spatial system adopted. Exogenous factors such as the value of travel time and a charging amount would play an important role in the degree of the response. Another important issue which should be borne in mind is that a more responsive model is not necessarily a better model.

Table 5-26 Results for the number of routes used by the household classes in the base run of the two models

O	D	The number of routes used			
		Single class	Multiclass		
			m_1	m_2	m_3
75	107	5	11	6	2
75	14	2	4	3	1
⋮	⋮	⋮	⋮	⋮	⋮

The multiclass bid-rent network equilibrium model can be a useful tool for policy tests. This is mainly because the multiclass model represents class-specific behaviour in the transport decision. As suggested in this chapter, the multiclass model generated the class-specific degree of responsiveness in terms of the social welfare changes and the elasticity values, whereas the single class counterpart suggested the aggregate average values. Thus, the multiclass bid-rent network equilibrium model was

advantageous in examining the effects responded to by different classes of population. Another example of the class-specific response can be found by examining the routes used by each class. Table 5-26 shows a part of the outcomes of the base run for the two models. As specified in the table, the multiclass model shows the class-specific number of used paths. Since link combinations for each route are predetermined before the algorithm runs, this output can be useful information for developing class-specific policy strategy towards an efficient urban structure.

To sum up, the broad results of the two models were similar. The similarity would imply that either approach could be used for investigating the interaction between transport and the location of activities. However, the multiclass bid-rent network equilibrium model would be more useful for policy tests in the sense that the multiclass model represents the class-specific degree of responsiveness.

5.9. Conclusion

This chapter has shown the results on the application of the bid-rent network equilibrium model. The overall purpose of this chapter was to demonstrate the ability of the model to represent the relationship between transport and the location of activities. This was considered in terms of analyses on the cause and effect of the spatial distribution. The city of Ansan in Korea was chosen for the case study. The test was conducted using the data of the Ansan transport master plan (1998) with some additional data. The simulation was undertaken in the single class model and its multiclass counterpart. The base run and the two policy runs were simulated in each model. The base run meant a simulation conducted using the data surveyed. The policy runs represented the introduction of a congestion charge and the release of land for housing development. Results were obtained in terms of the spatial distribution of households, which were discussed with respect to class-specific behaviour. The behaviour was demonstrated using the network performance indices of transport costs and locational benefits. The results are consistent with the design of the model, and satisfied common beliefs on the process of the land-use transport interactions. Thus, it could be concluded that the overall performance of the model would be satisfactory.

Nonetheless, it would be useful to note that long-term responses of this kind of model are strongly influenced by the relocation of economic activities. Unfortunately, investigations on the relocation are beyond the scope of this study. The quest would be possible in comprehensive land-use transport interaction studies. Hence, it is clear that there is plenty of scope for further work. The carrying out of that work would offer more opportunities for the analyses of the comprehensive land-use transport interactions. Despite this unsatisfactory aspect, the promising outcomes of the model would offer little doubt about the model proposed being a framework for investigating the relationship between transport and the location of activities.

6. Conclusions

This study has developed a bid-rent network equilibrium model. The model was suggested as an alternative framework for representing the nature of the relationship between transport and the location of activities. The model addressed the unique characteristics of locations using a hedonic interpretation. The compensatory decision and the best response competition under the framework of game theory systematically described the dual speculation of a household. The endogenous calculation of transport impedance and locational attractiveness was a successful approach to modelling of the interaction between transport and land-use. The three components were structured in a bi-level mathematical programme. The bid-rent network equilibrium model incorporated a multiclass framework in order to address interclass transport interactions. A heuristic algorithm solved the model reasonably. The operation was confirmed in numerical examples. The model was applied to a real network, which produced promising outcomes. The overall characteristics of the bid-rent network equilibrium model offer little doubt about the model being an alternative framework for investigating the relationship between transport and the location of activities. The key findings of this study responding to the research objectives that were specified in the introductory chapter can be summarised as follows:

In the **Introduction**, three essential components in the relationship between transport and the location of activities were identified. First, the unique characteristics of locations with regard to their heterogeneity and indivisibility were recognised as issues to be investigated. Secondly, decision-makers faced the interactions between decisions within a household and those between households. Thirdly, the mutual reaction between transport and land-use showed a cyclic relationship. The components were used as the nature of the relationship between transport and the location of activities in the development of the bid-rent network equilibrium model.

Chapter Two reviewed existing approaches classifying them into four major fields of models. Spatial interaction models were aggregate and deterministic, which meant the group failed to represent the unique characteristics of a location and the decision-making processes of a household. Furthermore, there was no explicit consideration of

the interaction between transport and land-use in this group. Mathematical programming models, owing to their aggregate nature, crudely addressed the systematic properties of a location and the behavioural context of a decision-maker. Combined models in this group showed a promising approach for analysing the interaction between transport and land-use, though the representation was partial. Random utility models treated a location as a bundle of attributes. The model offered a high degree of behavioural validity of a decision-maker. However, the interaction between transport and land-use was not explicit in this framework. Bid-rent models incorporated a hedonic theory to represent the heterogeneity and the indivisibility of locations. The decision-making process was considered using a bidding mechanism. However, it was not easy to establish the interaction between transport and land-use. Models may show interrelated outcomes. For example, the results of the spatial interaction model would be in agreement with those of the random utility model at the aggregate level; the Herbert-Stevens model could yield an analogous solution to the Alonso model provided that a planner had full information; the random utility and the bid-rent models might produce an identical market equilibrium in a perfectly competitive market. However, it should be borne in mind that none of the existing models, either independently or collectively, could satisfactorily represent the full nature of the relationship between transport and the location of activities.

The bid-rent network equilibrium model was developed in **Chapter Three**. The model was designed to satisfy the three characteristics of the relationship between transport and the location of activities. First of all, the hedonic interpretation of the model treated a location as being globally heterogeneous but consisting of homogenous elements. The interpretation reasonably recognised the two unique characteristics of locations. Secondly, game theory, accompanied by a bidding mechanism, represented the composite decision-making process of a household. The dual speculation was shown as an n-player non-cooperative game. Finally, the interaction between transport and land-use was realised as a mutual adjustment process towards a stable equilibrium. The adjustment produced the endogenous network performance indices of transport impedance and locational attractiveness. The three conceptual bases of the model were systematically embodied in a bi-level mathematical programme. The behaviour of households was translated into the problems of locators and non-locators. The decision-making process of locators was

considered using the hedonic-based random bid-rent model. The minimisation of the net interaction impedance was used to represent the behaviour of non-locators. The two problems were combined in the bi-level mathematical programme. The formulation was interpreted as equivalent to an oligopolistic Cournot game. The Nash equilibrium for this game was defined. The bid-rent network equilibrium model incorporated a multiclass framework, which systematically represented inter-class interactions on the transport network. The resulting formulation was interpreted as a seamless oligopolistic competition form of the Cournot game.

In **Chapter Four**, a heuristic algorithm for solving the bid-rent network equilibrium model was suggested. The algorithm combined a path-based routine for evaluating an equilibrium solution at the lower level with the Newton-Raphson procedure for estimating the parameters of the hedonic-based random bid-rent function in the upper level. The solution algorithm proposed had some attractions. First of all, the algorithm reasonably represented the behaviour of decision-makers in terms of the best responsive competition. Secondly, the resulting mutually consistent solution was compatible with the definition of the Nash equilibrium. Thirdly, the solution algorithm was simple in form and relatively easy to implement. Finally, the heuristic algorithm produced an acceptable solution in a reasonable computation time. The solution algorithm was successfully implemented in the numerical examples.

Chapter Five showed case studies of the bid-rent network equilibrium model. The overall purpose of this chapter was to demonstrate the ability of the model to represent the relationship between transport and the location of activities. A medium sized city in Korea was chosen. A base run and the two policy runs were simulated. The base run meant a simulation conducted using survey data. The policy runs represented the introduction of a congestion charge and the release of land for housing development. The spatial distribution of households was obtained, which was discussed in terms of the class-specific behaviour. The behaviour was demonstrated using the endogenous network performance indicators of transport impedance and locational attractiveness. The results were consistent with the design of the model and satisfied reasonable expectations of the process in land-use transport interactions.

Notwithstanding the successful modelling and the promising results from the bid-rent network equilibrium model, it is clear that there is plenty of scope for further work. Some representative areas can be discussed as follows:

(1) A trip table analysis: one of the principal outputs of the bid-rent network equilibrium model is a class-specific spatial interaction table. The table, in this study, represents the flow between residence and primary activity locations. The two locations of activities can be interpreted as origins and destinations respectively in travel demand analyses. This means that the bid-rent network equilibrium model can serve as a framework for trip table analyses.

(2) A tool for policy tests: the endogenous network performance indices in terms of transport costs and locational benefits are a distinct feature of the bid-rent network equilibrium model. In the policy runs of the case study, the model successfully evaluated the responses of the indicators invoked by some policy scenarios. The before-and-after references of the indices are the usual outcomes of policy tests. In particular, the bid-rent network equilibrium model suggests class-specific changes in demand and elasticity, generated by the changes in the endogenous indicators. These characteristics mean that the model is a useful tool for policy tests.

(3) Towards a comprehensive land-use transport model: long-term responses in the relationship between transport and land-use are strongly influenced by the relocation of economic activities. Unfortunately, investigations into these relocations were beyond the scope of this study. This extension of the model would mean the representation of multistage interactions between transport and diverse land-uses. In particular, the extension could involve the four types of locational behaviour that underlie urban dynamics. Thus, their incorporation would extend the model towards a comprehensive land-use transport interaction model. The carrying out of that work would offer more opportunities for the analysis of land-use transport interactions.

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Appendix I-Fundamentals of Game Theory

1. Introduction

This appendix describes the fundamentals of game theory. The bid-rent network equilibrium model has been developed using a game theoretical framework for representing the competition of households in the relationship between transport and the location of activities. The oligopolistic Cournot game, which is a special branch of n-player non-cooperative games, has supported the design of the model development. Even though the game theoretical interpretation in the development of the model has been satisfactorily specified, it would be useful to discuss some structures of game theory related to the framework adopted in this study. This is done in the appendix.

Since game theory is based on decision theory, this appendix starts with describing decision theory. Decision theory is divided into certainty and uncertainty cases in terms of the explicitness of ordering preferences. Subsequently, game theory is defined. Then, two forms of game representations are provided, namely normal and extensive forms. In the following section, the nature of game theory is investigated. Games are classified into cooperative and non-cooperative types. The non-cooperative games are subdivided into dominant strategy and best response games in terms of the characteristics of strategy. The Nash equilibrium is defined in the non-cooperative games under the best response strategy. In the next section, interpretations of applications of game theory to transport studies are examined. The interpretations are considered in relation to problem formulations of the applications.

The main body of this appendix is heavily dependent on two sets of lecture notes. One is the lecture notes on *Game Theory (2003)* by the Department of Economics, Massachusetts Institute of Technology (MIT), coordinated by Professor Muhamet Yildiz, and the other is the lecture notes on *Introduction to Game Theory (2003)* by the Department of Economics, Harvard University, coordinated by Professor Markus Mobius. The notes are downloadable from the web site of each department.

2. Decision theory

2.1. Introduction

As noted in the introduction, game theory is based on decision theory. Thus, it would be useful to investigate decision theory before reviewing game theory. Decision theory deals with situations in which agents have to make a choice. In this framework, the preference of one agent is assumed to be independent of the actions of the others. This means that no strategic interactions between agents are considered in decision theory, even though the theory deals with multiple players. Therefore, decision theory would be regarded as representing a single-player game theory. Decision theory can be divided into certainty and uncertainty cases in terms of the explicitness of ordering preferences. When there is an explicit order between outcomes, it is referred to as a certainty case; otherwise, it is called as an uncertainty case.

2.2. Under certainty

A decision problem under certainty (A, \preceq) consists of a finite set of outcomes A and a preference relation \preceq , e.g. $a \preceq b$ means that b is at least as good as a . The outcomes are assumed to be mutually exclusive, which means that an individual agent cannot choose two distinct alternatives at the same time. It is also supposed that a set of feasible outcomes is exhaustive so that the choice of a player is always defined. As for the preference relations, two self-evident definitions can be considered, namely *completeness* and *transitiveness*.

Definition 1. A relation \preceq is complete if and only if, given any two outcomes $a, b \in A$, either $a \preceq b$ or $a \succeq b$.

Definition 2. A relation \preceq is transitive if and only if, given any $a, b, c \in A$, $a \succeq b$ and $b \succeq c$ then $a \succeq c$.

These two axioms guarantee that all choices can be ordinal in a single chain with neither gaps nor cycles. The completeness represents the former, or no gaps, and the transitiveness shows the latter, namely no cycles. The two definitions suggest that any

relation can be a preference relation if and only if the relation satisfies the conditions of completeness and transitiveness.

A preference relation can be represented by a utility function $U : A \rightarrow \mathbb{R}$ as follows:

$$a \preceq b \text{ if and only if } U(a) \preceq U(b), \forall a, b \in A \quad (\text{I-1})$$

The utility function is called a VNM utility type because the representation was originated by von Neuman and Morgenstern (1944). The VNM utility function converts the decision problem easier because the function deals with physical values rather than an abstract preference relationship.

2.3. Under uncertainty

In decision problems under certainty, the preference relationship was explicitly ordered because the outcomes were finite. In contrast, the number of necessary comparisons is infinite in decision problems under uncertainty. This means that ordering preferences is difficult because the observation is, in principle, unobservable. In this case, the preference relation can be represented by probability.

Definition 3. A decision problem under uncertainty is defined as a set $A = \{(a_1, P_1), (a_2, P_2), \dots, (a_n, P_n)\}$ such that $\sum_{i=1}^n P_i = 1$ and $0 \leq P_i \leq 1$ where the outcome a_i occurs with probability P_i .

The probability of outcomes has no consistent rank among alternatives. The probability is only meaningful when it is represented in terms of ordinal values. The most well-known alternative approach to overcome this problem is von Neuman and Morgenstern' (1944) theory of an expected utility.

Definition 4. An expected utility of a decision problem under uncertainty $A = \{(a_1, P_1), (a_2, P_2), \dots, (a_n, P_n)\}$ is defined as $U(A) = \sum_{i=1}^n U(a_i) P_i$ where $\sum_{i=1}^n P_i = 1$ and $0 \leq P_i \leq 1$

A preference relation can be represented by von Neuman-Morgenstern's utility function. This VNM utility representation is supported by the assumption that a player

is a utility-maximiser. The assumption means that a player maximises their expected value of utility.

$$A \succeq B \text{ if and only if } U(A) \geq U(B) \quad (\text{I-2})$$

where A, B are distinct outcomes.

The necessary and sufficient conditions for the representation of the expected utility is, as is self-evident, the preference relation \succeq is complete and transitive. It is noted that a preference between utilities is represented by \geq rather than \succeq . This shows that the preference is ordered in terms of physical values.

3. Game theory

3.1. Definition

Decision theory, which was considered in the previous section, virtually dealt with a single decision-maker whose preference is assumed to be independent of the actions of other agents. In contrast, game theory represents a decision problem involving multiple persons where the preference of an agent is related to actions taken by the other agents. A definition of game theory can be given as follows:

Definition 5. Game theory is a formal way to analyse interactions among a group of rational agents who behave strategically.

Several important components for a framework of game theory can be observed in the definition. First of all, there is more than one decision-maker who is referred to as a player, namely $N = \{1, 2, \dots, n\}$. Each player is assumed to be rational so that an individual gamer chooses their best action to maximise payoffs. Secondly, the definition emphasises the interactions between players. Fundamentally, game theory assumes that the payoff of each agent is determined through the relationship between agents. In other words, the action of a player depends on actions available to each agent, each agent's preference on outcomes, and each player's speculation about the others' circumstances with respect to which actions are available to each player, how each player ranks outcomes, and their beliefs about the other players' beliefs. The

complex situation requires a strategy for a player to win a game. Again, this is because the action of a gamer is interdependent with other members of the group.

Definition 6. A strategy is a complete contingency-plan determining which action a player takes at each information set in which an agent is to move.

A player is assumed to have a finite set of alternatives of strategies

$$S_i = (s_1, \dots, s_k), \forall i \in N \quad (\text{I-3})$$

where S_i is a finite set of all strategies that are available to a player i and

s_k is the k^{th} pure strategy of a player i .

When a player knows exactly what strategies the other gamers are taking, an agent is believed to choose one definite strategy that can offer the maximum payoff. This case is referred as an agent playing a *pure strategy* game. However, in many cases, a player may not be exactly able to guess the other agents' actions. In this case, it is reasonable to suppose that a player draws a *mixed strategy*, combining a set of pure strategies with a probability distribution over the set of strategies.

$$S_i = \sum_k \sigma_k s_k = \sigma_1 s_1 + \dots + \sigma_k s_k, \forall i \in N \quad (\text{I-4})$$

where σ_k is the k^{th} non-negative real number satisfying $\sigma_k \geq 0$, $\sum_k \sigma_k = 1$ and

S_i is a mixed strategy of a player i ; a strategy profile of players is defined as a vector of mixed strategies of an individual player $S = (S_1, \dots, S_i, \dots, S_n)$.

Two important characteristics of the strategy are noted. First, a pure strategy can be interpreted as a special case of mixed strategies. If a player chooses one strategy with probability 1 and any one of the remaining strategies with probability 0, then the agent has effectively chosen one definite pure strategy. Secondly, since a mixed strategy for a player is determined by the probability of the player choosing the available pure strategies, the probability distribution for the non-negative values σ_k could be interpreted as the speculations of the player about the strategies of the others.

3.2. Representation of games

This section considers the way in which games are represented. In general, games are expressed in either a normal or an extensive form.

Definition 7. A normal form of an n -player game is a system $G = (N; S_1, \dots, S_n; U_1, \dots, U_n)$ where, for each player $i \in N = \{1, 2, \dots, n\}$, S_i is a set of strategies that are available to the player i and $U_i : \prod_i S_i \rightarrow \mathbb{R}$ is the player i 's von Neuman-Morgenstern utility function.

The utility function in the list G is represented by the von Neuman-Morgenstern type. Hence, an individual player is believed to maximise their payoffs. The payoff is determined by the process of strategic interactions between agents, which is represented in G by incorporating the list of the strategies of all gamers.

The normal form of game implicitly assumes that each player moves once and simultaneously. However, in this framework, neither the multiple movements of gamers nor a sequential decision-making process can be considered. An extensive form of game generalises the representation of games overcoming the drawbacks of the normal form. The extensive form introduces a tree that is defined as a set of nodes and directed edges connecting these nodes.

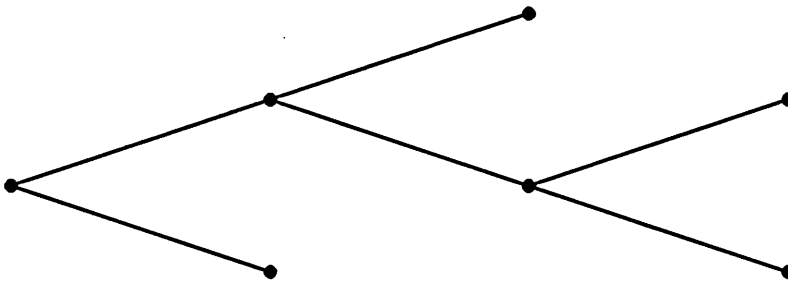


Figure I-1 A conceptual diagram of a tree in the extensive form of game

A typical structure of the tree is shown in Figure I-1. Each node has at most one incoming edge except the first node. For any two nodes, there is a unique path that connects the two nodes. A definition of the extensive form of game is given:

Definition 8. An extensive form of an n -player game is a system $G = (N; S_i; U_i; T)$ where N is a set of players, S_i is a set of available strategies

for each player i , U_i is the von Neuman-Morgenstern utility function of the player i , and T is the tree.

In the extensive form of game, each player is allocated to a specific node. The order of action is determined by the position of each individual player. The tree carries information set that is a collection of the position of a player, an order of movement, alternative successor nodes that are available to a player, and payoffs at each node.

4. Games and equilibriums

4.1. Introduction

There are many types of game that represent competition between players. Since each game has its own characteristics and a complex structure, it is impractical to classify games in terms of physical categories. Alternatively, it can be useful to impose some conceptual structures onto games for the purpose of analyses. The most widely used criteria for the classification are the characteristics of strategies and the relationship between players. This is because the two factors are basic components of games. In this study, games are understood as either a cooperative or a non-cooperative type in terms of the relationship between players. While the cooperative games allow the coalition of agents in the process of plays, the non-cooperative games assume no pre-play communication between players. The non-cooperative games are subdivided into dominant strategy and best response games. The sub-grouping is considered with respect to the characteristics of the strategy of games.

4.2. Cooperative games

Cooperative games allow players to have complete freedom of pre-play communication to make joint binding agreements. In these games, there is a finite set of players $N = \{1, 2, \dots, n\}$. Each player $i \in N$ receives an amount x_i that is the distribution of utilities available to the set of players in N . A payoff vector is denoted $\mathbf{x} = (x_1, \dots, x_n)$. Some players are assumed to coordinate strategies to maximise joint payoffs. This subset is called a coalition $g \subset N$. Coalitions include one-player

coalitions and void coalitions with no players at all. Payoffs to the players in the coalition g are determined by a characteristic function $v(g)$. The value of this function is assumed to guarantee the largest payoff to each coalition g . Two essential conditions should be met in order that games are formulated as the cooperative types.

Definition 9. Individual rationality is satisfied if and only if for each player i $x_i \geq v(i)$, $\forall i \in N$.

From the viewpoint of a player, participation in a coalition deserves consideration if and only if the coalition can guarantee higher payoffs than those of playing independently. No member in the coalition consents to receive fewer payoffs than those an agent can obtain by an independent play. Thus, the payoff of an independent action $v(i)$ should always be less than or equal to that of a joint playing x_i .

Definition 10. Group rationality is satisfied if and only if for each player i , $\sum_i x_i = v(N)$, $\forall i \in N$.

The definition of group rationality is common sense because the characteristic function $v(N)$ represents the maximum obtainable payoffs of players from the game. Otherwise, each player can gain without loss of the others, or exceed the amount at disposal. Thus, the condition can be interpreted as a special case of Pareto rationality.

Definition 11. A vector $\mathbf{x} = (x_1, \dots, x_n)$ that satisfies the conditions of individual and group rationalities is called imputation under the characteristic function.

An imputation represents a distribution of the available payoffs to an individual player. In this distribution, the two rationalities define the upper and lower bounds of the payoffs for a player. Then, using the imputation and the characteristic function, the cooperative game is defined as follows:

Definition 12. A system $G = (N, v, \mathbf{x})$ that consists of a set of players N , a characteristic function of this set $v(\cdot)$, and a set of imputations $\mathbf{x} = (x_1, \dots, x_n)$ satisfying the conditions of the individual and group rationalities is called a cooperative game.

Optimality principles for the cooperative games are diverse and the mechanism of solution finding is complex. The difficulty is because there could be a diverse level of partial agreements between players. Furthermore, sharing payoffs is not always possible when non-transferable units are involved. An investigation into this issue is beyond the scope of this study.

4.3. Non-cooperative games

Non-cooperative games meant competition in which no pre-play communication was allowed between players. It is essential to distinguish the non-cooperative games from the cooperative counterparts. The outcome of the cooperative games was determined by imputation. The individual payoffs were assigned as a result of an agreement among players rather than as a consequence of their actions. Therefore, the cooperative games emphasis the preference of agents in payoffs rather than situations that players face. Furthermore, a comparison of imputations is not limited to the individual payoffs but is more complex in nature. This is because the cooperative games consider coalitions. In contrast, the non-cooperative games are strategic games. The outcome of the non-cooperative games is formed as a result of the actions of those players in a particular situation. Therefore, the strategies of gamers play an essential role in determining the individual payoffs in the non-cooperative games. In this study, the non-cooperative games are subdivided into dominant strategy and best response games in terms of the characteristics of strategies.

4.3.1. Dominant strategy games

The dominant strategy game is based on the rationality and common sense of players.

Definition 13. A pure strategy s_i^ strictly dominates s_i if and only if $U_i(s_i^*, s_{-i}) > U_i(s_i, s_{-i}), \forall i \in N, \forall s_{-i} \in S_{-i}$ where $s_{-i} = (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$.*

The definition of dominant strategy means that no matter what the other players do, the strategy s_i^* is strictly better than s_i for a player i . In this case, it is common sense

that a player would never play the strictly dominated strategy s_i because a gamer is assumed to be rational.

Definition 14. A pure strategy s_i^ weakly dominates s_i if and only if $U_i(s_i^*, s_{-i}) \geq U_i(s_i, s_{-i}), \forall i \in N, \forall s_{-i} \in S_{-i}$ and $\exists s_{-i} \in S_{-i}, U_i(s_i^*, s_{-i}) > U_i(s_i, s_{-i})$.*

The weak dominance means that no matter what the other players do, the strategy s_i^* is at least as good as s_i , and there are some contingencies in which s_i^* is strictly better than s_i . When the strict dominance case is considered, it is common sense that a player would play s_i only if an agent believes that these contingencies will never occur. A cautious gamer who assigns some positive probability for each contingency will not play s_i .

An extension of the definition for the dominance of strategy under the pure strategy games to that of mixed strategy games is straightforward. The representation extended allows players to choose mixed strategies.

Definition 15. A mixed strategy S_i^ strictly dominates S_i if and only if $U_i(S_i^*, S_{-i}) > U_i(S_i, S_{-i}), \forall i \in N$ where $S_{-i} = (\dots, S_{i-1}, S_{i+1}, \dots)$.*

Definition 16. A mixed strategy S_i^ weakly dominates S_i if and only if $U_i(S_i^*, S_{-i}) \geq U_i(S_i, S_{-i}), \forall i \in N$ and $\exists S_{-i}, U_i(S_i^*, S_{-i}) > U_i(S_i, S_{-i})$.*

In conclusion, if a player is rational and has a strictly dominant strategy, then an agent always tries to play the dominant strategy game. If a player is cautious and has a weakly dominant strategy, then a gamer will not play other strategy games.

4.3.2. Best response games

In the best response games, there is no dominant strategy. This suggests that a rational player makes their best response with each other to achieve the maximum payoff.

Definition 17. For any player i , a pure strategy s_i^ is the best response to s_{-i} if and only if $U_i(s_i^*, s_{-i}) \geq U_i(s_i, s_{-i}), \forall i \in N, \forall s_i \in S_i$.*

Definition 18. For any player i , a mixed strategy S_i^ is the best response to S_{-i} if and only if $U_i(S_i^*, S_{-i}) \geq U_i(S_i, S_{-i}), \forall i \in N$.*

The definition of the best response strategy is the same as that of the dominant strategy except that the definition is represented in terms of the best response of a player against a specific strategy profile of the other gamers, namely s_{-i} or S_{-i} ; note that the dominant strategy is represented against all strategy set of the other players, namely $\forall s_{-i} \in S_{-i}$. If the best response strategy were preferable to all s_{-i} or S_{-i} , then the best response strategy would be a dominant strategy.

4.3.3. Equilibrium

A substantial part of an equilibrium theory for the non-cooperative games under the best response strategy was established by Nash (1950, 1951).

Definition 19. A strategy profile (s_1^, \dots, s_n^*) is pure strategy equilibrium if and only if for each $i, \forall i \in N$ the optimum strategy of the player s_i^* is the best response to the best strategies of the other players $s_{-i}^* = (s_1^*, \dots, s_{i-1}^*, s_{i+1}^*, \dots, s_n^*)$.*

Definition 20. For each player $i, \forall i \in N$, the Nash equilibrium of the pure strategy game is strict if and only if $U_i(s_i^, s_{-i}^*) > U_i(s_i, s_{-i}^*), \forall s_i \in S_i$.*

Definition 21. For each player $i, \forall i \in N$, the Nash equilibrium of the pure strategy game is weak if and only if $U_i(s_i^, s_{-i}^*) \geq U_i(s_i, s_{-i}^*), \forall s_i \in S_i$.*

At equilibrium, no player has an incentive to deviate from the equilibrium because the strategy is the best response to their beliefs about the strategies of the other players. If a strategy profile is dominant, then the profile is also the Nash equilibrium; but the reverse is not always true. An extension of the pure strategy equilibrium to the mixed strategy counterpart is straightforward.

Definition 22. A strategy profile (S_1^, \dots, S_n^*) is mixed strategy equilibrium if and only if for each $i, \forall i \in N$ the best strategy of the player S_i^* is the optimum response to the best strategies of the other players $S_{-i}^* = (S_1^*, \dots, S_{i-1}^*, S_{i+1}^*, \dots, S_n^*)$.*

Definition 23. For each player $i, \forall i \in N$, the Nash equilibrium of the mixed strategy game is strict if and only if $U_i(S_i^, S_{-i}^*) > U_i(S_i, S_{-i}^*)$.*

Definition 24. For each player $i, \forall i \in N$, the Nash equilibrium of the mixed strategy game is weak if and only if $U_i(S_i^, S_{-i}^*) \geq U_i(S_i, S_{-i}^*)$.*

Nash (1951) showed the existence of equilibrium states in any finite non-cooperative game under the mixed strategies.

Theorem 1. In any non-cooperative game under the mixed strategy, there is at least one equilibrium point.

Proof See Nash (1951).

5. Game theory in transport

This section deals with applications of game theory in transport studies. Game theory investigates various problems concerning conflict of interest by abstracting common strategic features. The theory has been recognised as a useful tool for modelling interactions between groups of decision-makers whose actions jointly determine outcomes (Fisk, 1984). Since the theory was adopted to interpret an equilibrium flow pattern as an n-person non-cooperative game (Dafermos and Sparrow, 1969), there have been extensive applications of game theory to transport studies.

Applications of game theory to transport studies are summarised in Table I-1. While non-cooperative games have been widely applied, an example of cooperative games can be found in the study of a highway cost allocation (Castaño-Pardo and Carcia-Diaz, 1995). The study used the Aumann-Shapley value of a non-atomic game supposing that the decisions of a single player are irrelevant to total outcomes. In this game, each platoon of vehicles is considered as a player and agents are assumed to co-work for a fair and rational cost allocation in the supply of highway facilities.

Table I-1 Applications of game theory to transport studies

Types	Examples
Non-cooperative games:	
n-person games	Hansen (1990) Garcia et al. (2000)
Oligopoly:	
Bertrand games	Yang et al. (2001)
Cournot games	Kita (1999) Bell (2000) Yang et al. (2001) Bell and Cassir (2002)
Stackelberg games	Yang and Bell (1998) Yang and Bell (2001)
Cooperative games	Castañó-Pardo and Carcia-Diaz (1995)

Applications of non-cooperative games can be divided in terms of the number of players. While games of oligopoly deal with a small number of players, the n-person games represent competitions of a large number of agents, though the number of gamers is supposed to be infinite. Since the games implicitly assume that players have no dominant strategy, the general Nash equilibrium under the best response strategy can be defined. The equivalent mathematical representation of the Nash equilibrium can be proposed as follows:

$$Y(S_i^*, S_{-i}^*) \geq Y(S_i, S_{-i}^*), \forall i \in N \quad (1.5)$$

where $Y(\cdot)$ is the payoff function of players;

S_i^* is the optimal mixed strategy of a player i ; and

S_{-i}^* is the optimal mixed strategy profile of all players except a player i ,

namely $S_{-i}^* = (S_1^*, \dots, S_{i-1}^*, S_{i+1}^*, \dots, S_n^*)$.

Interesting examples of n-person non-cooperative games are as follows. A model of the airline hub competition was formulated in terms of a n-player non-cooperative game between a set of airlines seeking to maximise profits (Hansen, 1990). The model found a state of quasi-equilibrium in which the round-to-round strategy adjustments by the airline competitors were small. Another illustration of the n-person non-cooperative game is found in the research into dynamic system optimal routings (Garcia et al., 2000). The study interpreted a routing mapping as the best reply of each

vehicle to routing decisions of the other vehicles in the system. A fictitious play represented an iterative procedure in which players, at each step, were supposed to make their best responses assuming the decisions of the counterparts would follow a historical probability distribution. In other words, players were assumed to compute the expected shortest paths supposing the other players were distributed according to the historical frequency of routing decisions.

The majority of the applications use an oligopolistic framework. Oligopoly represents interactions among small numbers of players who have conflict of interest. When there are only two players, this is called a duopoly. Three models of oligopolistic behaviour are found in the literature, namely a price-setting Bertrand model, a quantity-setting Cournot model, and a sequential quantity-setting Stackelberg model.

In the Bertrand competition, players compete in prices that firms simultaneously choose. In this game, products are assumed to be homogenous or identical no matter who produces them. Consumers are always believed to buy the product from firms that offer the lowest price. Therefore, each player addresses demand as follows:

$$D_i(\mathbf{p}) = \begin{cases} D(p_i) & \text{if } p_i < p_{-i}, \quad \forall i \in N \\ D(p_i)/n & \text{if } p_i = p_{-i}, \quad \forall i \in N \\ 0 & \text{if } p_i > p_{-i}, \quad \forall i \in N \end{cases} \quad (I-6)$$

where $D(\cdot)$ is a market demand function;

p_i is a market price that a player i sets; and

p_{-i} is the vector of market prices of players except an agent i , namely

$$p_{-i} = (\dots, p_{i-1}, p_{i+1}, \dots); \mathbf{p} = (p_1, \dots, p_i, \dots, p_n).$$

The Bertrand game is known to converge in an equilibrium in which firms set prices at the level of a marginal cost c . This can easily be demonstrated. When a player sets a price below the marginal cost $p_i < c$, the setting causes a loss. Hence, a reasonable player is likely to set the price at the price of rivals. The temporary price is unstable because some of the firms will reset their prices below the setting to maximise benefits. The change will motivate the remaining firms to adjust their prices up to the new setting. This process continues until no firm has an incentive to reset its price. In

the theory of microeconomics, the point is known to be the same level as the marginal cost where the firms make zero profits. When a firm sets its price higher than those of the other firms $c \leq p_{-i} < p_i$, the firm makes no profits. A reasonable player is likely to set the price equal to the price of rivals. The price converges in the marginal cost. Finally, when all firms set the same but higher price than the marginal cost $c < p_i = p_{-i}$, a reasonable player is likely to deviate from the setting to make bigger profits $p_i = p_{-i} - \varepsilon > c$ where ε is a positive small value. The movement makes rivals no profits, which motivates the other firms to adjust prices up to the price of the first-mover. The market price is determined at the level of the marginal cost. The price determined by firms is the point of the Nash equilibrium because no player has an incentive to deviate from equilibrium; if a player sets a higher price than the equilibrium price, the firm cannot sell any product, which makes zero profits; if a player sets a lower price than the equilibrium price, the price causes a loss.

Theorem 2. The Bertrand game has the unique Nash equilibrium

$$(p_i^*, p_{-i}^*) = (c, c), \forall i \in N.$$

Proof. See the above demonstration.

In general, firms in oligopoly make bigger profits than those in the perfectly competitive market because they can play as price-makers. However, in the Bertrand game, consumers play dominantly; consumers always choose a firm that supplies products at the lowest price. Firms should adjust prices up to the marginal cost in order to make profits. Thus, oligopolists act as if they were price-takers in the Bertrand game. This means that the outcome of the Bertrand game is the same as that of the perfectly competitive market. This is referred to as the *Bertrand Paradox*.

There are not many applications of the Bertrand game in transport studies. An interesting application of a mixed Bertrand-Cournot game can be found in the study of the competition between bus firms under the deregulated transport environments (Yang et al., 2001). The study formulated the types of bus firm as players that simultaneously choose their fleet size and frequency as well as fare to maximise profits; one has minibuses with a higher-fare and higher-quality service and the other has conventional buses with a lower-fare and lower-quality service. The study focused

on the effect of value-of-travel time distributions to the price and quantity competition between the two bus services. The research reported that equilibrium occurred when no player could increase their profits by changing service frequency and fare.

While firms in the Bertrand game simultaneously compete for prices, players in the Cournot game compete in quantities that agents choose simultaneously. In this game, there are small numbers of identical firms. Each firm is assumed to produce an homogenous product with a constant marginal cost; hence, players face an identical market price. Each gamer is assumed to adjust quantities to maximise profits. In this process, an individual gamer is believed to follow the best strategy in response to every other player since no player is supposed to have a dominant strategy. At equilibrium, no player has an incentive to deviate from equilibrium because the strategy is the best response to their beliefs about the strategies of the other players.

The Cournot equilibrium is one of the Nash equilibriums. This is simply checked as follows: the equilibrium is determined by the mutual reaction of players in terms of their best response to each other; players have no reason to alter their strategies at the equilibrium because the changes cannot improve the expected utility.

The framework of the Cournot game has been applied in several areas. The game was used to analyse the merging and yielding behaviour of cars in a ramp merging section (Kita, 1999). The study formulated the behaviour of merging and through cars as a two-person-non-cooperative game in the sense that each type attempts to take the best actions considering the best action of the other. Another example can be found in the study of the performance reliability of a transport network (Bell, 2000). The study proposed the framework of a two-player non-cooperative game between a group of network users and an evil entity: the network user was assumed to seek a path to minimise the expected cost; the hypothetical demon was supposed to choose link performance scenarios to maximise the expected cost. The relationship between them was regarded as non-cooperative in the sense that the user had no idea which link state would be invoked by the demon and the demon did not know which path the network user would choose. At equilibrium, the user could not reduce the expected transport impedance by changing his or her path choice probabilities and the demon could not increase the cost by changing the scenario probabilities, which was compatible with

the definition of the Nash equilibrium. An extension of this research (Bell, 2000) to the many-to-many case was made in Bell and Cassir (2002). The study included n-network users and m-OD specific demons, which represented a feedback between path choices and link costs; hence, the framework explicitly considered congestible network. A bi-level formulation was proposed to find a mutually consistent point. In the upper level, the demons maximised the total expected cost imposed on network users. In the lower level, a standard deterministic user equilibrium assignment problem was proposed from the viewpoint of network users.

There are similarities and dissimilarities between the Stackelberg and Cournot games. Both games deal with small numbers of gamers and investigate a quantity competition. In these games, no player has a dominant strategy. Hence, the best-response analysis can be applied to investigate the behaviour of decision-makers. The main difference between the two games is found in the order of gamers' actions. While the Cournot game assumes that all players act simultaneously, the Stackelberg game assumes a sequential decision-making process. In the Stackelberg game, one player acts before the others. Thus, the Stackelberg game can be understood as a best response game under an extensive form; the decisions of the players are made by the rule of the best response; the structure of a sequential decision-making process in the Stackelberg game is a special case of the frameworks of the tree. The first mover is referred to as a leader. The others are denoted as followers. For this reason, the Stackelberg game is referred to as a leader-follower game. The leader begins the game by announcing his or her decision. The followers execute their policies after the decision of the first mover. The leader tries to maximise his or her profits taking the reasonable reaction of the followers into account. In contrast, the followers simply react to the leader's choice, namely the best response to the leader's decision. Therefore, the leader has an advantage, which is referred to as the *first-mover advantage*. The equilibrium of the Stackelberg game can be expressed as follows:

$$\begin{aligned} \gamma(S_i^*, \Phi(S_{-i}^* | S_i^*)) &\geq \gamma(S_i, \Phi(S_{-i}^* | S_i)) \\ \text{and} \\ \gamma(S_i^*, \Phi(S_{-i}^* | S_i^*)) &\geq \gamma(S_i^*, \Phi(S_{-i} | S_i^*)) \end{aligned} \quad (I-7)$$

where a player i is the leader and $\Phi(\cdot)$ is the response function of the followers.

The Stackelberg equilibrium is one of the Nash equilibriums. The leader can only do worse by deviating from the equilibrium and the followers have no reason to deviate from the equilibrium because the followers play the best response to the leader.

The Stackelberg game has been rigorously applied in many transport areas. In particular, since the framework was interpreted as a useful tool for policy evaluations linked with a bi-level formulation (Bard, 1983), there have been comprehensive applications in the literature. For detailed examples, see Yang and Bell (1998, 2001).

6. Conclusion

This appendix has considered the fundamentals of game theory. The overview of game theory was intended to show the background framework of the design for the bid-rent network equilibrium model. The description started with representing decision theory. Then, game theory was considered. The review ended examining transport applications of game theory. Even though the game theoretical interpretations in the development of the model has been satisfactorily specified, this appendix would be useful to help understanding the conceptual basis of the model.

Appendix II-Inverse Transform Method

1. Introduction

In the simulation for the numerical examples in chapter four, the household data were generated using the inverse transform method. A simulation that considers stochastic components explicitly involves sampling or generating random variables. The process is associated with desirable probability distributions. While sampling is normally used for empirical applications of models, generation is applied to numerical illustrations of models. The generation of random variates means obtaining observations of probabilistic variables from a desired distribution. Several algorithms could be considered, but the inverse transform method was chosen in this study. The method has some attractive characteristics: first of all, the algorithm generates random variables with exactly the desired distribution; secondly, the method is efficient in terms of storage space and computation time; the efficiency is robust because the condition is satisfied not merely for some variables but for all parameters; finally, the inverse transform method facilitates the desired synchronisation and variance reduction. Because of these advantages, the technique is one of the most generally used algorithms (Law and Kelton, 1991) in the generation of probabilistic variates. In the next section, this appendix considers the general framework of the inverse transform method. The discussion is provided by both mathematical and graphical illustrations. This is followed by a description for the generation process of household data, which were used in the numerical examples of the bid-rent network equilibrium model. Finally, brief conclusions are presented.

2. The inverse transform method

It is noted that the discussion of the inverse transform method in this section is an assorted summary of Law and Kelton' description for the inverse transform method (Law and Kelton, 1991, pp. 465-474).

Let X be a random variate that is continuous. Let $F(\cdot)$ be a distribution function that is continuous and strictly increasing between the ranges $[0,1]$. This means that if

$x_1 < x_2$ and $0 < F(x_1) \leq F(x_2) < 1$, then $F(x_1) < F(x_2)$. Let $F^{-1}(\cdot)$ be the inverse function of the distribution function $F(\cdot)$. Then, the inverse transform method can be summarised in the two steps:

1. Generate $U \sim U(0,1)$ and
2. Return $X = F^{-1}(U)$

where ' \sim ' is read 'is distributed as'.

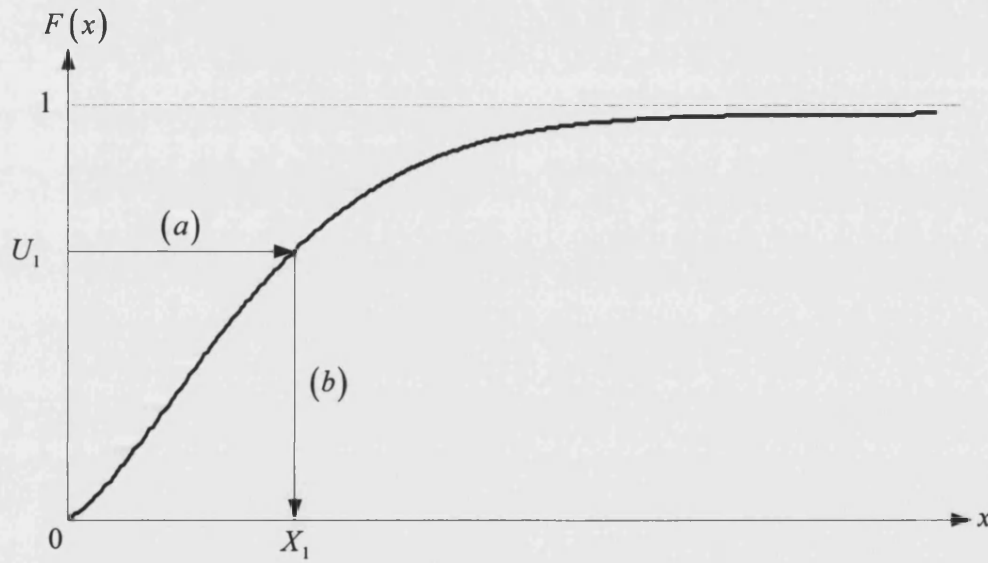


Figure II-1 A conceptual diagram of the inverse transform method

Figure II-1 shows a graphical illustration of the inverse transform method. First, the algorithm takes the random number U_1 , which is designed to be evenly spread on the interval $[0,1]$ in the vertical axis. Then, the method reads across (a) and down (b). The inverse function $F^{-1}(\cdot)$ is always defined because the value of both the random number U and the distribution function $F(\cdot)$ have the range $[0,1]$.

The value X generated has the desired distribution function $F(\cdot)$. Mathematically, this can be justified by observing that for any real number x , the probability that the random variate X takes a lower value than the number x is exactly $F(x)$.

$$P(X \leq x) = P(F^{-1}(U) \leq x) = P(U \leq F(x)) = F(x) \quad (\text{II.1})$$

This justification can be demonstrated graphically. Figure II-2 shows the Weibull distribution. Figure II-3 represents the density function of the Weibull distribution. In the diagrams, the shape parameter $\alpha = 1.5$ and the scale parameter $\beta = 6$. The density function would represent the relative chance of observing variates in different parts of the range. This means that many variates would be observed when the value of the density function is high. On the other hand, only a few variables would be observed when $f(x)$ is low. This relationship is easily confirmed in the diagram of the distribution function Figure II-2. The density function is the derivative of the distribution function, namely $f(x) = F'(x)$. This suggests that $f(x)$ is the slope function of $F(x)$. Hence, a steep gradient in $F(x)$ means more variates in $f(x)$ and a flat gradient in $F(x)$ means less variates in $f(x)$. In Figure II-2, a and b are selected in the steep and flat regions respectively. The ranges of a and b are designed to have the same size. This means that a and b have the same possibility of taking the random variates U . This is because the variates are assumed to be uniformly distributed. In spite of this setting, B has more space than A . This suggests that when the inverse transform method is considered, the random variates X would concentrate on the range A ; note that A is associated with the steep region a . Therefore, it can be concluded that the inverse transform method deforms the random uniform distribution $U(\cdot)$ into the desired distribution.

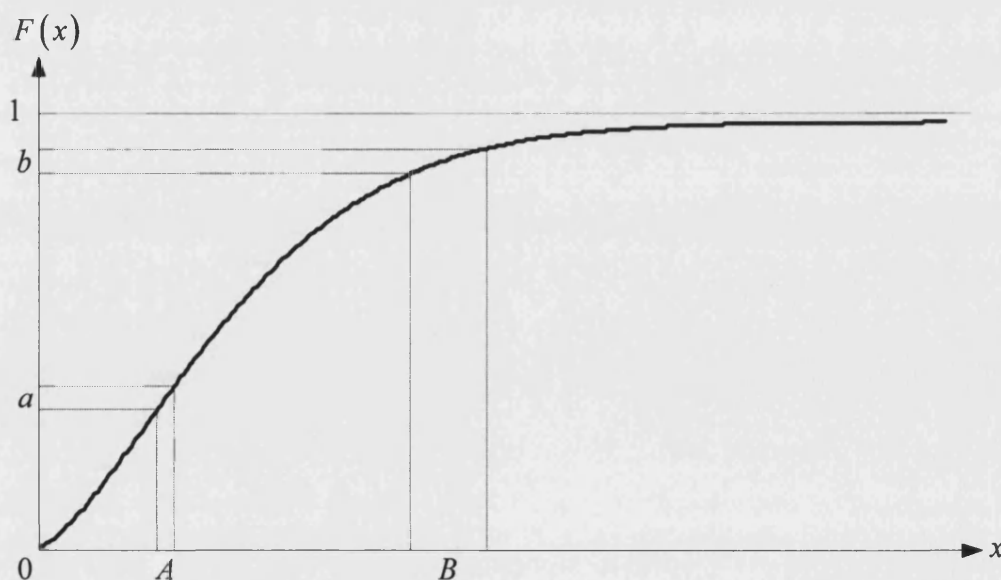


Figure II-2 The Weibull distribution (1.5, 6), source: Law and Kelton 1991, p. 467

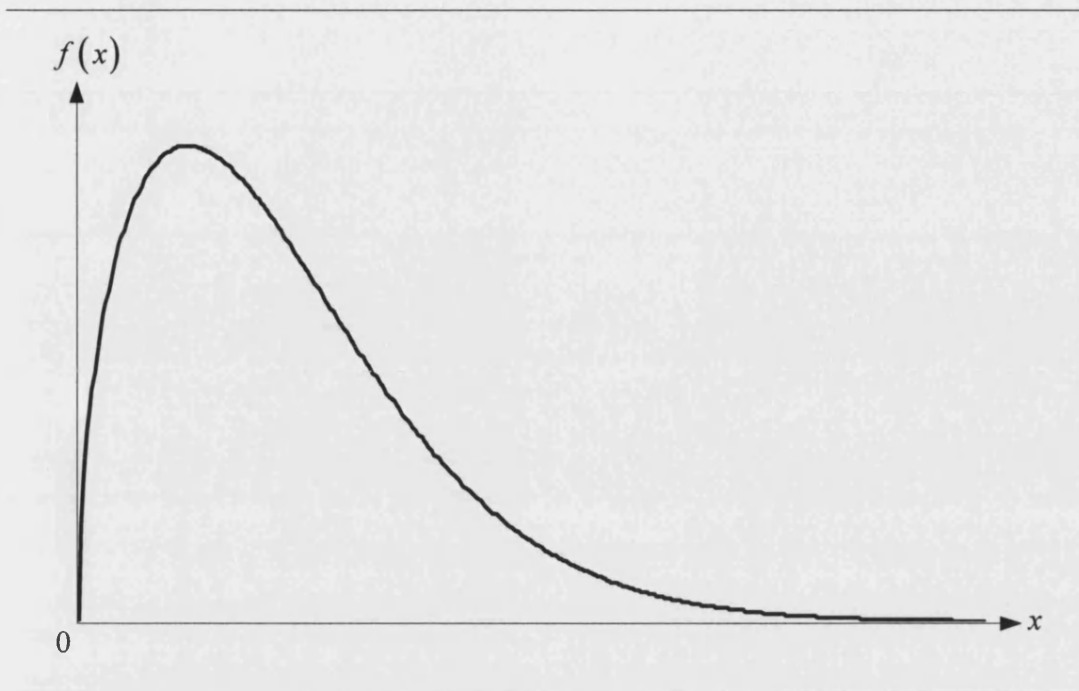


Figure II-3 The Weibull density function (1.5, 6), source: Law and Kelton 1991, p. 467

3. Generation process of the household data

The hedonic-based random bid-rent function in the upper level of the bid-rent network equilibrium model has assumed that the variates follow the Gumbel distribution. Since the natural logarithm of a Weibull random variable has a distribution known as the Gumbel distribution, the generation process considered the Weibull distribution in advance. Then, the values of the random variates were converted into the equivalent natural logarithm values. This section shows a technical summary of the process.

The Weibull distribution is given by

$$F(x) = \begin{cases} 1 - e^{-(x/\beta)^\alpha} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \quad (\text{II.2})$$

where $\alpha > 0$ is a shape parameter and $\beta > 0$ is a scale parameter.

The density function of the Weibull distribution is given by

$$f(x) = \begin{cases} \alpha \beta^{-\alpha} x^{\alpha-1} e^{-(x/\beta)^\alpha} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \quad (\text{II.3})$$

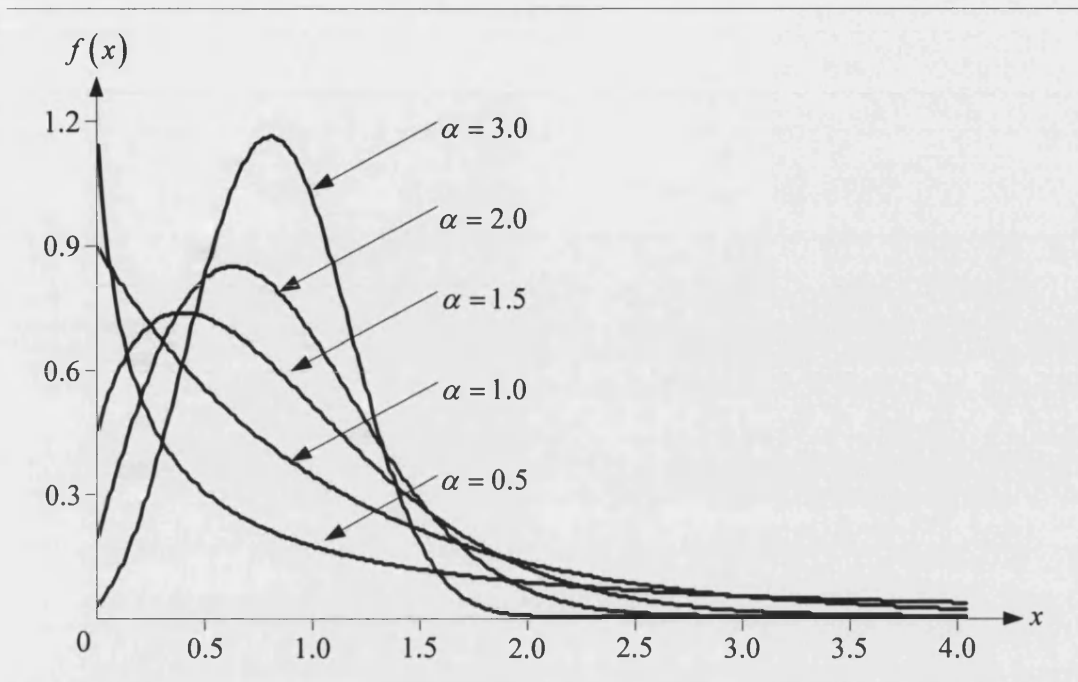


Figure II-4 Weibull density functions $(\alpha, 1)$

In the distribution and density functions, there are two parameters. The scale parameter changes the function of the distribution little. In the simulation, the value was set as $\beta = 1$. In contrast, the shape parameter causes considerable changes in the spread of the random variates. This suggests that the value of the shape parameter should be decided appropriate to the purpose of the simulation. The Weibull distribution was used in the generation of the random variates because the shape of the distribution is similar to that of the normal distribution, but execution time for the algorithm runs is much less than the normal distribution. Figure II-4 shows the Weibull distributions according to diverse shape parameters. When the value of the parameter is less than one, the shape of the Weibull distribution is far from that of the normal distribution. When α is greater than two, the deviation becomes bigger. Thus, it was decided to set the shape parameter as $\alpha = 2.0$ in the simulation.

The process of generating random variates is straightforward. First of all, the inverse function of the Weibull distribution is given by

$$F^{-1}(U) = \beta [-\ln(1-U)]^{1/\alpha} \quad (\text{II.4})$$

Secondly, the inverse transform method is applied.

1. Generate $U \sim U(0,1)$ and
2. Return $X = -\ln(1-U)^{1/2}$.

4. Conclusion

This appendix briefly overviewed the inverse transform method. The generation process of the household data, which were used for the numerical examples, was outlined. The description in this appendix is supplementary to help understanding the process of the simulation in chapter four.